DECISION FEEDBACK EQUALIZATION OF FIBRE OPTIC DIGITAL COMMUNICATION LINKS

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CERTIFICATE

Certified that this work on 'DECISION FEEDBACK EQUALIZATION OF FIBRE OPTIC DIGITAL COMMUNICATION LINKS' by Maj. S.K. Dogra has been carried out under our supervision and has not been submitted elsewhere for a degree.

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ABSTRACT

Performance of some fibre optic digital communication links with different source-detector combinations and fibre lengths are evaluated. Fibres are dispersive in nature and spread the optical signal on transmission through them. This dispersion with additive noise cause intersymbol interference (ISI) at high data rates. A decision feedback equaliser (DFE) has been used to reduce the effect of ISI.

To evaluate the performance of the DFE, the optical systems with different lengths and sources have been simulated on the digital computer DEC-1090 system. Simulations have also been carried out at much reduced data rates without the DFE. Results indicate that similar performance in terms of the probability of error of the system can be achieved without an equaliser, but, at much reduced data rates. P(e) better than 1.25×10^{-7} has been obtained with laser source and P(e) of 2.5×10^{-5} with LED source for a SNR of 14 dB.

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CHAPTER 1

INTRODUCTION

1.1 GENERAL:

The use of light for sending message information goes far back into time. In 1876 Alexander Graham Bell [1] explored the use of light for audio Communication. The invention of the LASER [2] in 1960-61 stimulated widespread interest in the use of light for communication purpose. In 1966 K.C. Kao and G.A. Hockham [3] proposed that optical fibre be used as dielectric wave-guides in under ground cable to carry information. The advent of optical fibres added a new dimension to optical communication. In 1968 typical fibre losses were above 1000 dB/km, but by 1970 losses under 20 dB/km were achieved.

Presently installed fibres have losses in the range of 3 dB/km or less at wave lengths of 820-850 nm. Laboratory samples have shown losses as low as 0.47 dB/km at 1550 nm wavelength. The present objective is to develop fibres with losses around 1 dB/km for commercial use.

The advantages offered by fibre optical communication are enumerated below.

a) The system is immune to EMI, RFI and cross talk.

b) It offers large bandwidth and low loss.

c) Small size and weight with consequent ease of installation and reduced transportation volume.

d) Potential low cost

e) No electrical ground loop or short circuit problem

f) Reduction of total system power consumption. Actual power output used is of the order of mw.

1.2 OPTICAL FIBRE COMMUNICATION SYSTEM:

The block schematic of the fibre optic digital link is shown in Fig. 1.1. The system can be divided into two main parts - one part which deals with the electrical signals and the other part dealing with the optical energy. The later part is shown within the dotted lines in the figure and includes the optical source, the optical fibre and the photodetector.

The different subsystems in the fibre optical digital link are briefly explained below.

<u>Driver:</u> Like in any communication system the input signal or data first modulates a carrier which is propagated through the channel. Here too the input modulates the optica

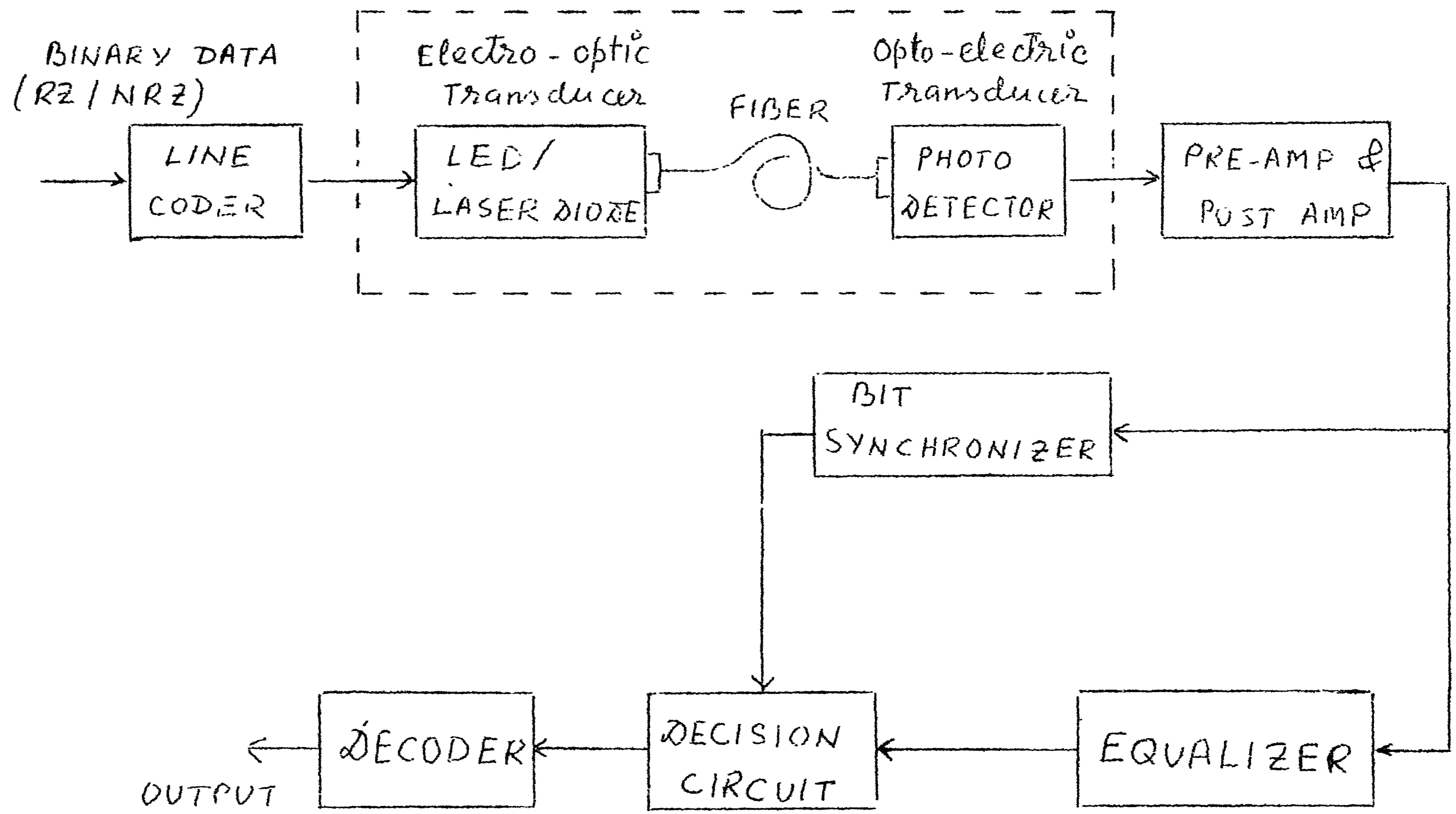

FIG 1.1 OPTICAL FIBRE COMMUNICATION SYSTEM

source directly, when the input is '1', the optical source is turned on otherwise it is kept off.

Electro-optical Transducer: Semiconductor laser and light emission diode (LED's) are being used for converting electrical signals into optical signals. In the system under discussion we have used both the laser diode as well LED.

Optical Fibre: The optical signals are propagated through the optical fibres channel. These signals undergo some losses and distortion depending upon the characteristic of the fibre. The distortion is such that it spreads the signal pulse in time domain. This is mainly caused by two types of dispersion in optical fibres, i.e. intermodal and material dispersion. However, in graded-index fibres only the material dispersion is significant.

Opto-Electric Transducer: The optical signal is received by a photo-detector, which converts the incident optical signal into an electrical signal. The photodetectors are mainly of two types.

1. P-I-N Diodes
2. Avalanche photodiodes (APD's)

Front End Receiver: The front end receiver circuit is a combination of a pre-amplifier and a post-amplifier. As

the data rate is very high, the pre-amplifier is implemented using a FET, CA-3127E.

Equalizer: The distortion in the optical fibre channel spreads the signal pulse in time domain. The material dispersion and the additive noise of the system cause ISI. An equalizer is used to reduce the effect of the ISI. In the system under discussion we have used a Decision feed-back equalizer, which reduces the magnitudes of the interfering samples.

Bit Synchroniser: The bit synchroniser recovers the symbol timing clock from the received signal and selects the sampling instants. The equalized signal is sampled at the right instants and at the rate at which it is received. A decision is then made about the sampled signal using a threshold detector, and we get the required signal after it is decoded suitably.

In any communication system the input signal modulates a carrier which is propagated through a channel. Here we consider the input as a Binary data sequence of '1' or '0'. The optical source output intensity is modulated by the input electrical signal. Thus when a '1' is transmitted the optical signal having some intensity propagates through the fibre. The impulse response of the fibre

spreads the signal pulse in time domain. The attenuated and dispersed optical signal when arrives at the receiving end of the fibre, it is converted into an electrical signal by direct detection in the photodetector. During the detection process certain noise inherent to the detection mechanism and th front-end circuits get added to the signal. The effect of the noise depends upon the type of the photo detector used. This will be discussed in a later section.

By the principle of superposition for optical fibre transmission (Personick [4]) single pulse description can be extended to obtain a model for the transmission of an entire sequence of ON and OFF pulses. The photo detector output r(t), can be written as

$$r(t) = \sum_{K} g_K \, s(t-t_K) + n(t) \tag{1.1}$$

where the time instants t_K form a poisson process having intensity $\lambda(t)$, with

$$\lambda(t) = \sum_{i} a_i \, p_F(t-iT) + \lambda_0 \tag{1.2}$$

where g_K = avalanche gain factor

$s(t)$ = output pulse of photodetector

$n(t)$ = additive noise

T = signalling interval

$\lambda_o \geq 0$ is the dark current

$a_i = 0,1$ equiprobable data symbols

$p_F(t) \geq 0$ pulse response of the fibre

The output from the photodetector as given by eq. (1.1) is then filtered to reduce the effect of noise by passing through a low noise pre-amplifier. A decision is then made about the received signal using a threshold detector and we get the required signal.

An optical fibre communication system might differ from the one discussed in this section depending upon the source, data rates and fibre length used. These aspects have been discussed in the chapters to follow.

1.3 NOISE IN PHOTO-DETECTORS:

The photo-detector current consists of the sum of displacement currents of individual hole and electron pairs generated within the photo-detector. The time at which electrons are generated due to absorption of the impinging photons are random and obey Poisson distribution. Every individual optical pulse (correspondence to '1') differs from the average by some unpredictable amount and this difference can be termed as a signal dependent noise. In practice electrons are also emitted when no

optical signal (photons) fall on the photo-detector. This is called the dark current and is modelled by introducing a constant additive intensity function λ_o before the detector (see eq. (1.2)). To transmit a '0', the optical pulse is not propagated for that bit interval, but, the detector emits electrons due to thermal ionization (dark current).

Another source of noise is thermal noise which is always present in electronic circuits. A circuit model (Fig. 1.2) is shown to illustrate the various noise sources. The output current from the photo-detector is given by

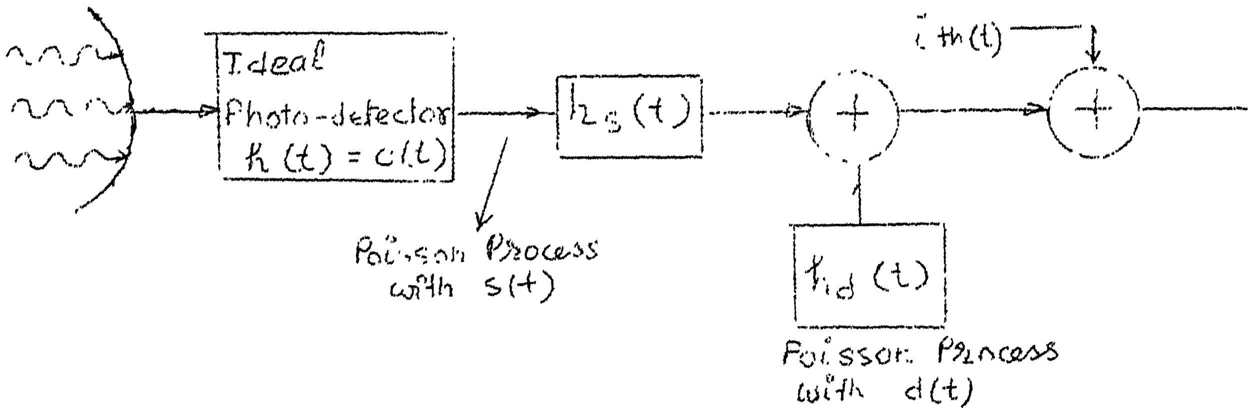

FIG 1·2 DETECTOR MODEL

$$i(t) = i^s(t) + i^d(t) + i^{th}(t) \tag{1.3}$$

where,

$$i^s(t) = \sum_{j=1}^{N_s} g_j\ h_s(t-z_j)$$

$$I^d(t) = \sum_{j=1}^{N_d(t,t+T)} g_j\ h_d(t-z_j)$$

where $i^s(t)$ is the shot noise process (signal dependent) and $N_s(t,t+T)$, the number of photo-electrons generated in a closed interval $[t,t+T]$, is random and has Poisson distribution with mean $\lambda_s(t)$

where $\lambda_s(t) = \int_t^{t+T} p(t)\ dt$

z_j is the time when the jth electron is emitted, $h_s(t)$ is the time invariant filter modelling the component function of the shot noise. $i^d(t)$ is the dark current process with an intensity λ_o,

$i^{th}(t)$ is the additive thermal noise process and

g_j is the gain of the photo diode.

In the case of a PIN diode detector $g_j=1$, and pulse signal-to-noise ratio, $\int h_s^2(t)/N_o$, is of the order of -20 dB. Therefore, the shot noise is neglected in comparison with the thermal noise in the case of PIN diode.

In the case of APD detector the gain is large. This increased g_j, increases the shot noise. Pulse signal-to-noise ratio $\int g^{-2} h_s^{2}(t)dt/N_o$ is of the order of 20 dB. Here the shot noise is quite large compared to the thermal noise and hence thermal noise is neglected when APD detector are used. As we have used a PIN diode detector in the F.O.C. system studied, we can neglect the shot noise component.

1.4 EQUALIZATION:

The model of the F.O.C. system has been discussed in Section 1.2. In that section we have noticed that received pulses are spread out in time. Therefore, the transmission of higher and higher data rates will be possible if the fibre optic medium accomodates larger BW. Otherwise, with the higher data rates, ISI will increase and inturn increasing the probability of error.

There are two general methods for equalization:

a) Frequency domain equalization, and

b) Time domain equalization

Here we will discuss Time-domain equalization. Since early 1960, there has been a lot of interest in time domain equalizers.

A typical fibre optic receiver for digital signalling schemes consists of a photo-detector, an amplifier, an equalizer and a decision device, along with the necessary synchronization circuits as shown earlier. We treat the fibre optic channel as linear in power [4,5], i.e.

$$P(t) = \sum_{i=-\infty}^{\infty} I(i)\ h_p(t-iT) \tag{1.6}$$

where,

$P(t)$ - is optical power driving the photo-detector

$I(i)$ - optical power from source

$h_p(t)$ - is the fibre response

The average signal at the equalizer output is

$$< y(t) > = \sum_{i=-\infty}^{\infty} I(i)\ p(t-iT) \tag{1.7}$$

where $p(t)$ is the overall pulse response. The equalizer output is given by

$$y(t) = \sum_{i=-\infty}^{\infty} I(i)\ p(t-iT)+n(t) \tag{1.8}$$

where $n(t)$ is noise, modelled as a Gaussian-plus filtered-Poisson process.

In time domain, the received signal after the photo-detector is fed into the equalizer which is a delay line with multiple taps, viz. Transversal filter of Fig. 1.3. The tap weight vector $C \triangleq [C_1 C_2 \ldots C_n]^T$ is

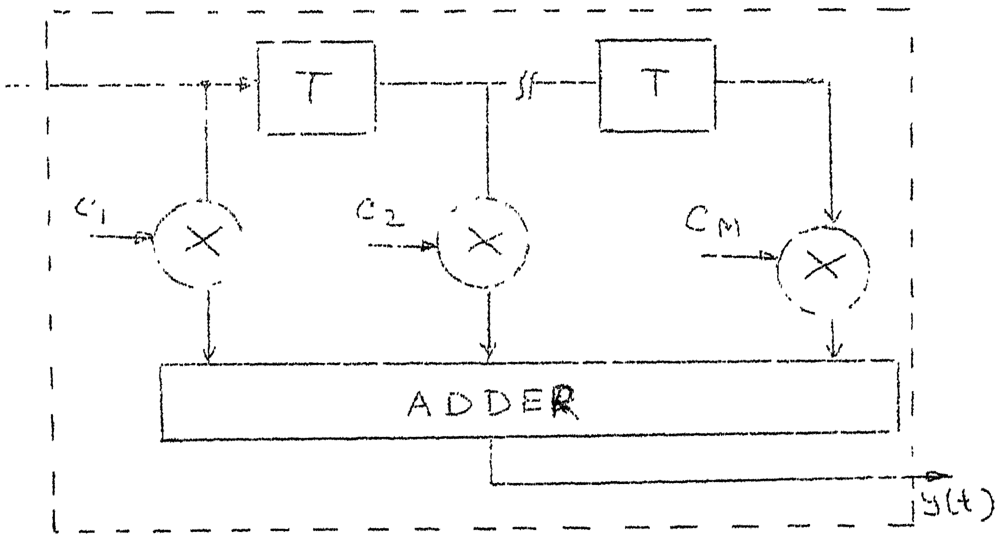

FIG 1·3 TAPPED DELAY-LINE EQUALIZER

computed in order to satisfy a suitable criteria. Two equalization criteria are often used, viz. zero-forcing as suggested by Lucky [6] and minimization of the mean-square error (MSE) [7]. In the zero forcing criterion the output distortion 'D' is minimized, where

$$D = \frac{1}{q_0} \sum_{K=-\infty,\ K \neq 0}^{\infty} |q_K|$$

and the peak distortion of the unequalized pulse < 1.

In first case, C is computed by solving a linear system. The second criterion minimizes the MSE between the received sequence estimate and the transmitted sequence. From a mathematical view point, this approach searches for

linear least squares solution. The main draw back of the tapped delay line equalizer results from the fact that the length of the equalizer tends to infinity.

In 1967 Austin [8] determined a new suboptimal receiver structure which is non-linear and uses its previous decisions to reduce ISI in high speed data transmission. It is called a Decision Feedback Equalizer (DFE) as it feeds back the output symbols through another TDL for comparison with the forward TDL output. After this a lot of work has been done on DFE. The main advantages of DFE are:

i) Number of taps required for the forward TDL is much less than the transversal filter equalizer (unitary equalizer) tap delay line.

ii) The number of taps required for the backward TDL is also limited.

iii) The performance of the DFE is better than the unitary equalizer for the same data rate.

1.5 EQUALISATION IN FIBRE OPTIC DIGITAL TRANSMISSION:

Since the development of fibre optic digital communication, a lot of work has been done for the minimization of ISI and noise.

Foschini et.al. [9] in October 1975 worked on optimum direct detection of digital data signal. In their

approach they processed the output of the photo-detector, so that the probability of error is minimum using the Viterbi algorithm detection principle. Viterbi algorithm detector is difficult to implement in hardware for high data rate systems with present day technology. A sub-optimal solution using a DFE as the first stage in place of the first VA detector of above has been considered by Govind Sharma [10]. The second stage of the receiver as obtained by him is also quite complex as it requires large memory and a very high sampling rate.

Dogliotti et.al. [11] in 1976 considered the problem of base-band equalization in optical PCM fibre system. For the given data and fibre characteristics they have suggested an equalizer transversal filter, where the tap weight vector $C \triangleq [C_1, C_2 \ldots C_n]$ are computed in order to satisfy a suitable criterion for the minimization of ISI. In this paper as well as one by Dogliotti and A. Luvison [12], the authors have minimized ISI by sequence estimation technique. In such a case, the received process is written according to its 'state equations' and the optimum estimator is derived as a Kalman Filter [13]. For TDL structures, the MMSE criterion guarantees better performance as shown in Fig. 1.4. This figure corresponds to a probability of error of 10^{-9} and shows that MMSE

equalization is better than zero forcing equalization and also when $\sigma/T<1$, then we can even ignore the equalizer provided that SNR is high enough.

D.G. Messerschmitt [14] considered a digital fibre optic transmission system with Poisson signal statistics and additive wide sense stationary noise. The criterion of optimization is MSE between the decision threshold input and the current data digit S_K. He has considered the zero forcing criterion as well as MMSE for both linear and non-linear (DFE) equalizer and obtained solution for each criterion. Some special cases like white noise and band limited noise have also been considered. Capt. R. Wahi [15] designed and evaluated the performance of the DFE based on the Austin [8] sub-optimal receiver structures. He also included predistortion filter in the transmitter and then evaluated the performances of system. At lower SNR values pre-distortion filter reduces error and gives a gain of approximately 1.5 dB. G. Tamburelli [16] has examined the performances of decision feedback and feed-forward receiver where the post-cursor correction is carried out only once while the precursor correction takes place many times without requiring complex circuits as the relevant transversal filter consists of only one or two taps. He has shown that the post-cursor of the

incoming signal is compensated by the decision feedback, thus at the output of a first decision circuit the transmitted signals are evaluated with an error probability relevant to a decision feedback receiver, and precursor are compensated by other decision circuits which follow the first. He has shown that this arrangement is superior to the existing ones and allows a transmission rate equal to twice the Nyquist rate.

1.6 PROBLEM DESCRIPTION:

The problem considered in this thesis is to evaluate the performance of the Digital Fibre Optical Communication System at different data rates. The following systems are considered.

a) Source-laser diode, link length 2.155 km

b) Source-LED, link length 2.155 km

c) Source-laser diode, link length 4.310 km

At higher data rates, noise and ISI affect the performance of the system. This can be reduced with the help of an equalizer. We have used a DFE for this purpose. The tap weight values for the DFE are calculated by Monsen's Adoptive linear filtering method.

The F.O.C. system is simulated on the digital computer DEC-1090 system to find the maximum data rate that can be transmitted over the system with and without the DFE for different SNR values.

1.7 ORGANISATION OF THESIS:

This thesis comprises 5 chapters. In Chapter 2, we have briefly reviewed the work done on adaptive equalizers by Peter Monsen. Chapter 3 describes a fibre optic communication system and its link parameters and the design of the DFE. It also describes the impulse responses for various link lengths and the bandwidth of the channel.

Chapter 4 describes the simulation of the DFE on the DEC 1090 system. Tap gain settings and the probability of error for various SNR's at different data rates are tabulated.

Chapter 5 concludes this thesis with a discussion of the results obtained, and some suggestions for possible future work.

CHAPTER 2

DECISION FEEDBACK EQUALIZER

2.1 INTRODUCTION:

Synchronous digital communication systems are degraded primarily by noise and intersymbol interference. To combat these effects, equalization techniques [17],[18], have been suggested. In 1967 Austin [8] considered a feedback receiver that compensated for the effects of intersymbol interference due to both future and past symbols. His optimum structure consists of a matched-filter, transversal filter combination (forward filter) operating on the received signal and a feedback linear filter operating on the past decisions. The past decisions have been assumed to be correct. Non-stationary effects of the channel were not considered.

Peter Monsen [19] has worked on adaptive equalization again under the fundamental assumption of no decision error so that the output digits can be assumed correct. The optimality criterion is minimum-mean-square error (MMSE). Under this criterion, the receiver is shown to be superior to one without the feedback path.

This feedback receiver, besides mitigating noise and ISI acts to eliminate timing jitter and Doppler shifts in the channel.

Some theoretical calculations and experimental results indicate that the error propagation effect due to incorrect decision is not large. (Austin has also shown it). Since the ISI span is normally short, a discrete Markov process model for analysis of the error propagation effect can be used to determine the error propagation effect.

2.2 SYSTEM DISCRIPTION:

The communication model under study considers transmission of serial binary data over a linear channel for which the dispersion has width of the order of the data rate reciprocal. The channel dispersion produces ISI which can be excessive.

The communication model is shown in Fig. 2.1. If the

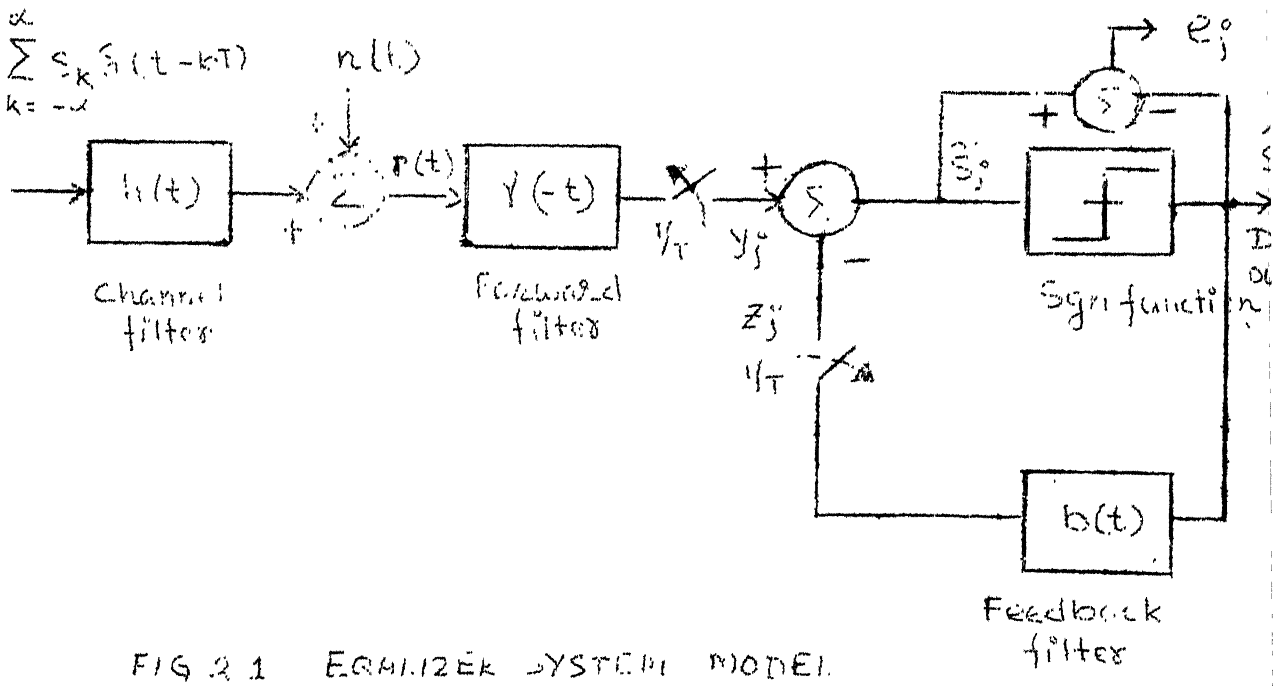

FIG 2.1 EQUALIZER SYSTEM MODEL

channel impulse response is $h(t)$, the received signal $r(t)$ is given by

$$r(t) = \sum_i S_i h(t - iT) + n(t) \tag{2.1}$$

where T is the interval between symbols.

The binary digits S_i are equally likely, and have a correlation function.

$$C_K = E \{S_i S_{i+K}\} \tag{2.2}$$

The noise term is a realization of a zero-mean white Gaussian process with spectral density μ.

The receiver consists of two linear filters, the forward filter that operates on the received signal, and the backword filter that operates on the reconstructed data. Delays in the channel impulse response can force the use of a noncasual forward filter when the transmission delay is normalized to zero. Accordingly, the impulse response of the forward filter $\gamma(-t)^*$ is defined over the entire range $-\infty < t < \infty$.

The backward filter has discrete input and output and thus can be realized as a transversal filter with symbol interval tap spacings. Because the filter is located in a

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*It is notationally convenient to define the forward filter impulse response as the function $\gamma(-t)$.

feedback loop, it can only operate on past reconstructed digits. Its impulse response can then be written in terms of the transversal filter tap gain b_i as

$$b(t) = \sum_{i=1}^{\infty} b_i \delta(t-iT) \tag{2.3}$$

The reconstructed digits are formed by a decision device that produces the sign of the analog sample at the summed output of the two filters.

2.3 EQUALIZATION FOR KNOWN CHANNEL:

To solve the time varying digital communication problem, various optimization criteria will be investigated for a stationary environment. After considering the different criteria, adaptive techniques will be considered. The input of the forward corrective filter is the sum of the signal and noise terms.

$$r(t) = \sum_{K=-\infty}^{\infty} S_K\, h(t-KT) + n(t) \tag{2.4}$$

The forward filter linearly operates on the reconstructed discrete synchronous output. The outputs of the forward and backward filters are combined. The impulse response of the backward filter can be written as

$$b(t) = \sum_{K=-\infty}^{\infty} b_K \delta(t-KT) \tag{2.5}$$

Thus the combined signal into the decision device at the sampling time jT is given by

$$\hat{S}_j = Y_j - Z_j = \sum_{j=-\infty}^{\infty} \gamma(u)\; r(jT+u)du - \sum_{\ell=1}^{\infty} b_\ell \; S_{j-\ell} \tag{2.6}$$

Under the assumption $\hat{S}_K = S_K$, the input to the decision device can be split into the signal, noise and intersymbol interference terms. These components found from equations (2.4) and (2.6) are as follows

$$\hat{S}_j|_{\text{signal}} = S_j\, \underline{\gamma}'\underline{h}_o \tag{2.7}$$

$$\hat{S}_j|_{\substack{\text{intersymbol}\\ \text{interference}}} = \sum_{\ell\neq o} S_{j-\ell}\, \underline{\gamma}'\underline{h}_\ell - \sum_{\ell=1}^{\infty} b_\ell \; S_{j-\ell} \tag{2.8}$$

$$\hat{S}_j|_{\text{noise}} = \underline{\gamma}'\underline{n}_j \tag{2.9}$$

A receiver design depends upon the criterion which minimises the intersymbol interference term and the noise term.

2.3.1 Unitary Equalizer:

In this section following types of unitary equalizers are described in brief.

1. Transversal Filter Equalizer
2. Zero Forcing Equalizer

2.3.1.1 Transversal Filter Equalizer:

For a unitary equalizer in a receiver consisting of only a forward filter and a decision device, the inter-symbol interference term can be eliminated if a function $\gamma(t)$ can be found. $\gamma(t)$ may be a tandem configuration.

$$\gamma(-t) = \int_{-\infty}^{\infty} g(-u)\, W(t-u)\,du \tag{2.10}$$

where $g(-t)$ is the impulse response of a fixed filter with bandwidth roughly equal to that of the channel, and $W(t)$ is the impulse response of a transversal filter with tap spacing equal to the symbol interval.

If the transversal filter has N_1 negative - index tap outputs and N_2 positive index tap outputs, $W(t)$ takes the form

$$W(t) = \sum_{i=-N_1}^{N_2} W_i\, \delta(t-iT) \tag{2.11}$$

where W_i is the ith tap gain. These tap gains are then to be chosen to minimise the effects of intersymbol interference. The forward filter impulse response is given by

$$\gamma(-t) = \sum_{i=-N_1}^{N_2} W_i\, g(iT-t) \tag{2.12}$$

and the system pulse response is

$$q_K \triangleq q(KT) = \underline{\gamma}'\,\underline{h}_K = \sum_{i=-N_1}^{N_2} W_i \int_{-\infty}^{\infty} g(u+iT)h(u+KT)du \tag{2.13}$$

The integral in eqn. (2.13) is the pulse response at the input to the transversal filter at the sampling time (K−i)T. Let

$$\theta_K = \int_{-\infty}^{\infty} g(u)\, h(u+KT)du \tag{2.14}$$

Thus eqn. (2.13) can be written as

$$q_K = \sum_{i=-N_1}^{N_2} W_i\, \theta_{K-i} = \underline{W}'\, \underline{\theta}_K \tag{2.15}$$

where,

$$\underline{W} = \begin{vmatrix} \underline{W}_{N_1} \\ \\ W_{N_2} \end{vmatrix} , \quad \underline{\theta}_K = \begin{vmatrix} \theta_{K+N_1} \\ \\ \theta_{K-N_2} \end{vmatrix} \tag{2.16}$$

For the unitary equalizer the intersymbol interference term, eqn. (2.8), is then

$$\dot{S}_j \Big|_{\substack{\text{inter symbol}\\ \text{interference}}} = \sum_{l \neq 0} q_l\, S_{j-l} \tag{2.17}$$

with a finite number of degrees of equalization freedom. It may not be possible to force eqn. (2.17) to be identically zero. A possible optimization problem is

$$\min \sum_{k \neq 0} q_k^2 \quad , \; q_0 = \text{constant} \tag{2.18}$$

The performance of the unitary equalizer can be considered by first evaluating the performance when N_1 and N_2 are infinity and then investigating the case when N_1 or N_2 are restricted to a finite number of degrees of freedom. When N_1 and N_2 are infinity, the ISI can in most cases be reduced to zero if

$$q_K = \sum_{i=-\infty}^{\infty} W_i \theta_{K-i} = 0, \quad K \neq 0$$

2.3.1.2 Zero Forcing Equalizer:

Intersymbol interference can also be minimised by a zero forcing equalizer as suggested by Lucky [6]. The peak distortion for the pulse $q(t)$ is

$$D = \frac{1}{|q_0|} \sum_{K=-\infty}^{\infty} |q_K| \tag{2.19}$$

The input pulse to the equalizer will be assumed to have initial peak distortion D_0. The reference sample x_0 will also be normalized to unity. The problem is to determine the tap gain coefficients for a tap transversal equalizer which minimises the final peak distortion.

Lucky establishes that at minimum distortion, at least L-1 values of q_K must be zero. This gives L-1 equations

in L-1 unknowns, but the question remains as to which L-1 zeros in the output response does one force to achieve minimum distortion. A key theorem is then proved which shows that if the initial distortion

$$D_o = \frac{1}{|\Theta_o|} \sum_{\substack{K-\infty \\ K \neq o}}^{\infty} |\Theta_o| \qquad (2.20)$$

is less than unity then the minimum output distortion must occur for those L-1 weight values which simultaneously cause the corresponding output response sample to be zero, i.e.

$$q_K = 0 \quad K \in K_L,\ K \neq 0 \qquad (2.21)$$

when $D_o > 1$, the settings which force corresponding output zeros may not be the optimum values, and a distortion greater than the minimum may result.

This approach is designed for channels where not only is intersymbol interference the limiting factor (since noise power is not considered) but, also where the intersymbol interference is bounded above. This situation exists to a large extend on telephones channels where the technique has been applied.

2.3.2 Feedback Equalizer:

Last section described the class of unitary equalizers which operate on only the received signal. The feedback

equalizer also considers the reconstructed data sequence $\hat{S}_K$, which has been assumed to be the transmitted data. This section describes the choice for forward and backward filters for an intersymbol interference minimization. The prefilter is taken as a matched filter so the input noise power to the transversal filter is minimized.

From eq. (2.8) the ISI term at the input to the decision device is

$$\tilde{S}_j \Big|_{\substack{\text{inter symbol} \\ \text{interference}}} = \sum_{\ell \neq 0} q_\ell \, S_{j-\ell} - \sum_{\ell=1}^{\infty} b_\ell \, S_{j-\ell} \tag{2.22}$$

where

$$q = \sum_{i=-N_1}^{N_2} W_i \, \phi_{\ell-i} \tag{2.23}$$

For minimization of intersymbol interference, the feedback filter be

$$b_K^+ = q_K = \sum_j W_j \, \phi_{K-j}, \quad K = 1,2,\ldots,\infty \tag{2.24}$$

For this choice of a feedback filter, all the ISI due to past pulses is conceled exactly. The ISI power becomes

$$Y^2 = \left\{ \sum_{K=-\infty}^{-1} q_K \, S_{j-K} \right\}^2 \tag{2.25}$$

which, with an infinite number of degrees of freedom can be reduced to zero. The optimum weights solve the semi-infinite set of equations.

$$\sum_{i=-\infty}^{\infty} W_i^{+} \phi_{K-i} = 0, \quad K < 0 \tag{2.26}$$

Peter Monsen [19], solved these equations by spectrum factorization.

Since unitary and feedback equalizers reduce the ISI to zero, one would qualitatively expect the feedback equalizer to be superior because of its noise advantage.

2.3.3 Mean-square Error Minimization:

For channels which are not limited solely by either ISI or additive noise, an optimizing criterion should be chosen which minimises both the effects. The feedback receiver minimises the mean square error. The error is defined as the difference between the analog voltage at the input to the decision device and the corresponding transmitted binary digit, i.e.

$$e_K = \hat{S}_K - S_K \tag{2.27}$$

where

$$\hat{S}_K = \sum_{\ell} S_{K-\ell}\, h'_{\ell}\, \gamma - \sum_{\ell=1}^{\infty} S_{K-\ell}\, b_{\ell} + \gamma'\, n_K \tag{2.28}$$

Here error is produced by both ISI and additive noise. The minimum mean-square-error equalizer minimizes the mean

square of eqn. (2.27) with respect to both forward and backward filters. By including the backward filter in the optimization. Monsen has shown that the MMSE feedback equalizer is superior in the absence of decision error to the MMSE unitary equalizer.

This criterion of minimum-mean-square error (MMSE) has been compared with the minimum probability of error (MPE) criterion. The later is the best criterion for the class of receivers that consider only one symbol at a time. The MPE receiver and its associated probability of error were determined by using steepest descent techniques. It was seen that the MPE receiver had significantly better performance than the MMSE receiver. However, it was finally concluded from this study that although the two criteria are not completely equivalent in performance, a vast majority of situations do lead to equivalent performance. The simplicity in the use of the MMSE criterion is then the deciding factor in the choice of the criterion.

The minimisation problem to be studied can be stated as

$$\min_{\gamma, b_i} \text{MSE} = \overline{(\hat{S}_K - S_K)^2}, \quad i = 1,2,\ldots \infty \qquad (2.29)$$

The input to the forward filter in the vector notation will be defined as $r_K = r(t+KT)$, so that

$$\underline{r}_K = \sum_{\ell} S_{K-\ell}\, \underline{h}_{\ell} + \underline{n}_K \tag{2.30}$$

$$\hat{S}_K = \underline{r}_K' \underline{\gamma} - \sum_{\ell=1}^{\infty} S_{K-\ell}\, b_{\ell} \tag{2.31}$$

Then let

$$\underline{S}_K = \begin{vmatrix} S_{K-1} \\ S_{K-2} \\ \cdot \\ \cdot \\ \cdot \end{vmatrix} \quad , \quad \underline{b} = \begin{vmatrix} b_1 \\ b_2 \\ \cdot \\ \cdot \\ \cdot \end{vmatrix} \tag{2.32}$$

be semi-infinite column vectors, so the decision device input is

$$\hat{S}_K = \underline{r}_K' \underline{\gamma} - \underline{S}_K' \underline{b} \tag{2.33}$$

The MMSE optimization is invariant to the choice of time origin so that for simplicity the zeroth sampling instant is chosen. The minimization can now be cast into a more familiar form, i.e.,

$$\min_{\underline{\gamma}, \underline{b}} \text{MSE} = \overline{(\underline{r}' \underline{\gamma} - \underline{S}' \underline{b} - S)^2} \tag{2.34}$$

By the introduction of partitioned vectors, eqn. (2.34) can be cast in the functional form of a single vector variable, as follows:

$$\min_{\mathbf{f}} \text{MSE} = (\underline{f}' \underline{V}_K - S_K)^2 \tag{2.35}$$

where

$$\underline{f} = \begin{bmatrix} \gamma \\ \cdots \\ \underline{b} \end{bmatrix}, \quad V_K = \begin{bmatrix} r_K \\ \cdots \\ -S_K \end{bmatrix} \tag{2.36}$$

The solution of eqn. (2.35) leads to the solution of the set of equations.

$$< V_K \; V_K' > f = \; < V_K \; S_K > \tag{2.37}$$

where the < > brackets are employed to denoted the expected value.

The solution for the forward filter impulse response is found to be of the form

$$\gamma(-t) = \sum_i W_i h(iT-t) \tag{2.38}$$

which is the representation for a matched filter in tandem with a transversal filter with symbol-interval tap spacing. The transversal-filter impulse response is written in terms of the tap gains W_i as

$$W(t) = \sum_i W_i \, \delta(t-iT) \tag{2.39}$$

The receiver is then specified by the tap gain set $\{W_i\}$ $i=-\infty$ to ∞ associated with the forward filter and the tap gain set $\{b_i\}$ $i=1$ t associated with the backward transversal filter. The solution for these tap gains were determined by Monsen in

terms of their 'Z' transform. The technique of spectrum factorization [25] was employed to determine the solution for W(z) and B(z). The solutions are

$$W(z) = [U^{-}(z)\ U^{+}(o)]^{-1} \tag{2.40}$$

$$B(z) = T[U^{+}(z)/U^{+}(o)]^{-1} \tag{2.41}$$

The positive superscript indicates a spectrum factor with all its poles and zeros outside the unit circle, whereas the negative superscript factor has them all inside. Since W(z) has no singularities out side the unit circle, it follows that the forward tap-gain function is anticausal i.e.

$$W_i = o \quad i > o$$

The forward transversal filter thus operates only on future values of the matched filter output.

Thus, in the absence of decision errors, the intersymbol interference dual to past digits is cancelled exactly by the backward filter if the source digits are uncorrelated. Under this condition the forward filter then minimizes the mean-square error due to noise and the ISI due to the future digits. It is interesting to compare the optimum filter and MMSE for the feedback and nonfeedback cases. The non-feedback or unitary equalizer (UE) is

a special case of the feedback equalizer (FE), namely, when B(f) is set identically to zero. Thus the optimum FE is superior in a MMSE sense, to the optimum UE.

2.3.4 A Gradient Technique Realization:

For the fixed known channel, a steepest-descent or gradient technique was found for the ISI minimization equalizer. For the MMSE receiver, a gradient technique would need to synthesize three separate filters, namely, the matched filter, forward filter and the backward filter.

In any real system the channel will be band limited effectively, so the channel and the matched filter can be specified in sampled-data form.

The received signal r(t) in sampled data form is

$$r(t+KT) = \sum_{i=-\infty}^{\infty} r\left[\frac{i}{B} + KT\right] \operatorname{sinc}(Bt-i) \qquad (2.53)$$

The optimum solutions are identical to the one obtained for the mean square error in Section 2.3.3. The difference is that the forward filter can now be synthesized by one transversal filter with tap spacings equal to the Nyquist interval (1/B).

A gradient algorithm for a MMSE receiver takes the form

$$\underline{f}_{n+1} = \underline{f}_n - \alpha\,[\overline{\underline{V}\,\underline{V}'\underline{f}_n} - \overline{\underline{V}_n\,\underline{S}_n}\,] \qquad (2.54)$$

where $\underset{\sim}{f}_n$ is the receiver vector (2.36) at the nth iteration. Eqn. (2.54) can be cast into the form :

$$\underset{\sim}{f}_{n+1} = \underset{\sim}{f}_n - \alpha \underset{\sim}{\nabla} \overline{e_n} \qquad (2.55)$$

where e_n is the error at the nth iteration. The gradient technique and the adaptive receiver implementation using this techniques are discussed in the next chapter.

2.4 ERROR PROPAGATION EFFECT:

The DFE uses receiver decision in an attempt to cancel ISI due to the past pulses. A fundamental assumption has been made that the reconstructed data sequence is identical with the transmitted data. In reality it is not true. Decision errors will occur and they will cause two effects. The first is that of an adaptation error. Since the reconstructed data sequence is employed as a test signal to determine the learning algorithm parameter. Decision error will further perturb the ideal equalizer tap gain setting. The second effect is more severe.

The DFE decisions are linearly filtered in order to develop the appropriate voltage for the ISI cancellation. When the decision is wrong, the probability of decision error in the immediate future is enhanced as the decision error propagate through the backward filter. This effect is thus termed Error Propagation. The resulting error

propagation will be larger than the error propagation under the No decision error premise.

Monsen [20] analysed the magnification of the error probability due to the Error propagation effect for a single echo channel and a truncated geometric response and observed that degradation due to error propagation effect is very small.

Since the error propagation effect is negligible, the DFE is superior to the linear equalizer. Moreover, because the backward filter eliminates interference due to all the past digits, the number of transversal filter taps in the forward filter can be reduced. This leads to an implementation advantage and improved adaptation as the self noise due to adaptation increases with the number of taps.

CHAPTER 3

D.F.E. FOR FIBRE OPTIC LINK

3.1 INTRODUCTION:

An important area of application for optical fibres is the transmission of high speed digital data signals. Even though the dispersion effects caused by a fibre are greatly reduced with respect to other kind of channels, the main limiting factor in increasing the data rate is imperfect channel characteristics.

The approach to match the exact characteristics of the system to reduce the effect of noise and ISI is not practical since it is difficult to design transmitting and receiving filters having the exactoptimal characteristics. We have seen that the DFE is a simpler alternative which provides a considerable reduction of the effects of noise and ISI. In the next section adaptive DFE based on the gradient technique has been described.

3.2 ADAPTIVE DFE:

For a fixed known channel, an equalizer realized by gradient technique minimises the mean-square error. This technique is used to synthesize three separate filters, i.e. the matched filter, the forward filter and the backward

filter. Furthermore, the matched filter impulse response is a continuous time function. In any real system the channel will be effectively band limited to some region $|f| \leq B/2$. So the channel and the matched filter can be specified in sampled-dat. form. The continuous vector $\gamma = \{\gamma(t)\}_{t=-\infty}^{\infty}$ can then be replaced by the infinite-dimension discrete-index vector.

$$\gamma = \left\{ \gamma_i = \gamma(i/B) \right\}_{i=-\infty}^{\infty} \tag{3.1}$$

and γ completely describes $\gamma(t)$ by the interpolation formula

$$\gamma(t) = \sum_{i=-\infty}^{\infty} \gamma_i \operatorname{sinc}(Bt-i) \tag{3.2}$$

If the channel is band limited to $B/2$, it is reasonable to position a filter prior to the receiver which will also band limit the additive noise to the range $|f| \leq B/2$. Then the received signal $r(t)$ also has a sampled data representation of the form

$$r(t+KT) = \sum_{i=-\infty}^{\infty} r[\tfrac{i}{\beta} + KT] \operatorname{sinc}(Bt-i)$$

So the vector $\underline{r}_K = \left\{ r(t+KT) \right\}_{i=-\infty}^{\infty}$ can be replaced by

$$\underline{r}_K = \left\{ r[\tfrac{i}{B} + KT] \right\}_{i=-\infty}^{\infty} \tag{3.3}$$

The optimization problem is identical and if infinite-length

transversal filters are used to realize the forward and backward filters, the optimum solutions correspond to the transfer function solutions determined in Chapter 2. The difference is that the forward filter can now be synthesized by one transversal filter with tap spacing equal to the Nyquist interval 1/B.

The optimum solution determined from eqn. (2.37) depends on the correlation matrix

$$\overline{\underline{V}_K\ \underline{V}'_K} = \begin{bmatrix} \overline{\underline{r}_K\ \underline{r}'_K} & -\overline{\underline{r}_K\ \underline{s}'_K} \\ -\overline{\underline{s}_K\ \underline{r}'_K} & \overline{\underline{s}_K\ \underline{s}'_K} \end{bmatrix} \tag{3.4}$$

being non-singular. The quadratic form associated with eqn. (3.4) is

$$QF = \underline{u}'\,\overline{\underline{r}_K\ \underline{r}'_K}\,\underline{u} - 2\underline{u}'\,\overline{\underline{r}_K\underline{s}'_K}\,\underline{w} + \underline{w}'\,\overline{\underline{s}_K\ \underline{s}'_K}\,\underline{w}$$

$$= \overline{(\underline{u}'\ \underline{r}_K - \underline{w}'\,\underline{s}_K)^2} \geq 0 \tag{3.5}$$

Thus $\overline{\underline{V}_K\ \underline{V}'_K}$ is at least non-negative definite.

To prove positive definiteness, consider the random process

$$A_K = \sum_{i=-\infty}^{\infty} u_i r[KT+i/B] - \sum_{i=-\infty}^{\infty} w_i s_{K-i} \tag{3.6}$$

or equivalently

$$A_K = \underline{u}' r_K - \underline{W}' S_K$$

This random process is stationary since both of its component processes are stationary. If the power spectrum is non-zero in any frequency region for arbitrary $||\underline{u}||^2 + ||\underline{W}||^2 > 0$, QF is strictly positive and eqn. (3.4) is positive definite. The process r(t) has an additive noise component with non-zero spectrum over the entire band of interest. Also the source correlation in the real problem is finite in extent and the source spectrum has energy at all frequencies. Thus the QF must be positive unless the process $\underline{r}_K$ and $\underline{S}_K$ are linearly related. But $\underline{r}_K$ has a noise component which is independent of the source digits, so a linear relation of that form cannot exist. Thus $\overline{\underline{V}_K\ \underline{V}_K'}$ is positive definite.

The positive definiteness of $\overline{\underline{V}_K\ \underline{V}_K'}$ not only guarantees that the inverse matrix solution to eqn. (2.37) exists, but also insures that a gradient algorithm will converge if the step size α is chosen small enough. A gradient algorithm for a MMSE receiver takes the form

$$\underline{f}_{n+1} = \underline{f}_n - \alpha\ [\overline{\underline{V}\underline{V}'}\underline{f}_n - \overline{\underline{V}_n\ \underline{S}_n}\ ] \qquad (3.7)$$

where $\underline{f}_n$ is the received vector at the nth iteration and the term in brackets is equal to one half the gradient with

respect to $\underline{f}_n$ of the MSE expression (2.34). From the expression for the error process, (3.7) can be cast into

$$\underline{f}_{n+1} = \underline{f}_n - \alpha\, \overline{\underline{V}\, e_n} \qquad (3.8)$$

In terms of the forward and backward filter components, eqn. (3.8) becomes

$$\underline{\gamma}_{n+1} = \underline{\gamma}_n - \overline{\text{[illegible]}_n} \qquad (3.9)$$

$$\underline{b}_{n+1} = \underline{b}_n + \alpha\, \overline{\underline{S}\, e_n} \qquad (3.10)$$

The vector $\underline{r}$ represents the input to the forward filter and $\underline{S}$ represents the input to the backward filter. The components of $\underline{r}$ and $\underline{S}$ are the tap values of the respective forward and backward transversal filters. Thus the simple algorithms (3.9) and (3.10) are implemented by simply correlating the tap values with the error process and incrementing the corresponding tap gains. From the analysis in Chapter 2 the algorithms will converge if α is chosen so that

$$\rho(\overline{\underline{V}\, \underline{V}'}) < \frac{2}{\alpha} \qquad (3.11)$$

where $\rho(.)$ represents the largest magnitude eigenvalue of its argument matrix. In equation (3.8) tap gains are updated after reception of few say L digits.

The algorithm (3.8) has been analysed by Monsen [19] who determined convergence conditions, the performance neighborhood and convergence rate. The performance neighbourhood is measured by a quantity M called misadjustment which is the ratio of the steady state excess mean-square error to the MMSE, i.e.

$$M = \lim_{n\to\infty} \frac{1}{\beta} \langle (f_n' V_n - S_n)^2 \rangle$$

where the MMSE is $\beta = 1 - \langle VS\rangle' \langle VV'\rangle^{-1} \langle VS\rangle$. For good data performance it is generally necessary to have a small misadjustment, i.e. $M \ll 1$.

Monsen showed that the best choice of averaging interval is L = 1, i.e., no averaging at all. Thus the algorithm reduces to

$$\underline{f}_{n+1} = \underline{f}_n - \alpha \, \underline{V}_n \, e_n \tag{3.11}$$

which interms of forward and backward taps become

$$\underline{\gamma}_{n+1} = \underline{\gamma}_n - \alpha \, \underline{r}_n \, e_n \tag{3.12}$$

$$\underline{b}_{n+1} = \underline{b}_n + \alpha \, \underline{\hat{S}}_n \, e_n \tag{3.13}$$

The forward filter is transversal filter with tap spacing equal to 1/B where B/2 is the channel bandwidth. The forward filter output at time nT is given by

$$Y_n = \int_{-\infty}^{\infty} \gamma_n(t)r(nT+t)dt = \frac{1}{B} \sum_{i=-\infty}^{\infty} \gamma_n\left[\frac{i}{B}\right] r\left[nT+\frac{i}{B}\right]$$

(3.14)

which can be written as

$$Y_n = \underline{\gamma}_n' \underline{r}_n \tag{3.15}$$

The forward-filter output has the output of the backward filter subtracted from it. The backward filter is also a transversal filter with tap spacing T and tap gains b_i. Its input is the reconstructed data sequence $\{\hat{S}_K\}$ which is assumed to approximate the transmitted sequence $\{S_K\}$.

The backward-filter output can be written as

$$Z_n = \sum_i b_i \hat{S}_{n-i} = \underline{b}' \hat{\underline{S}}_n \tag{3.16}$$

Thus the filter tap gains are updated by simply incrementing the present value as a fraction of the product of the tap voltage and error sample at that instant. The error voltage at time nT is given by

$$e_n = Y_n - Z_n - \hat{S}_n \tag{3.17}$$

The receiver configuration just described is shown in Fig. 3.1. In addition to reducing noise and intersymbol

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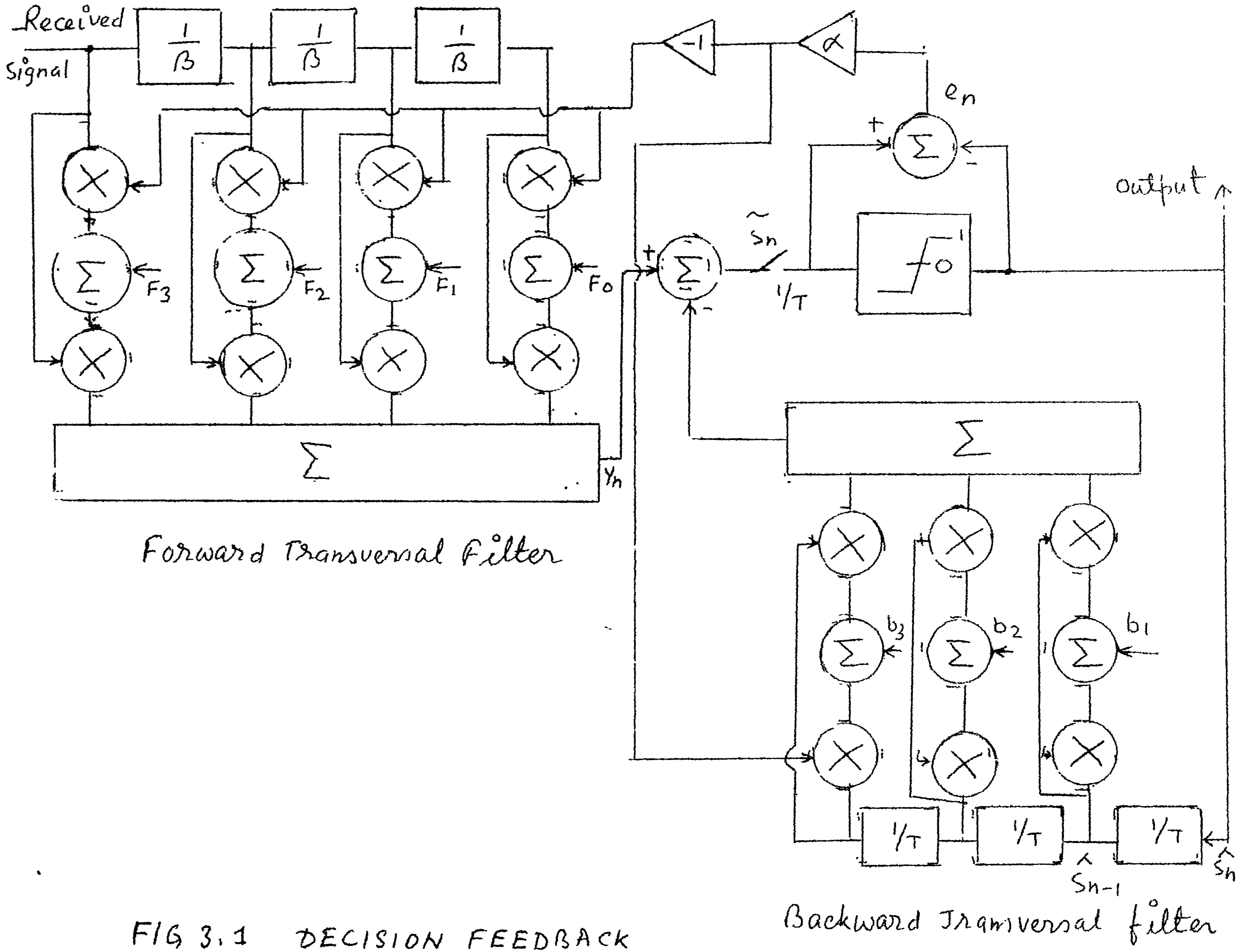


FIG 3.1 DECISION FEEDBACK EQUALIZER

interference, the receiver has some additional advantages. Since the forward-filter tap spacing is equal to the Nyquist interval, the algorithm can adapt to changes in transmission delay. Thus, timing jitter introduced by changes in transmission delay can be eliminated because the delay variations are less than the adaptation rate required.

3.3 DATA RATE:

The fibre transmission medium has a finite dispersion for the propagating optical pulse as discussed in Chapter 1, and a finite loss rate for the optical transmitted power. These both limit the data rate that can be transmitted as the photo-detector requires a certain minimum signal power for a given error probability and data rate. Semiconductor laser or LED sources have limited output power of the order of 2 mw for laser and 100 μw for LED that can be coupled into the fibre. These facts taken together restrict the fibre length one wishes to interface between the source and detector.

The Fig. 3.2 [21] shows the receiver sensitivity for PIN and APD detector digital receivers for various bit rates. The figure was calculated for an error rate of the order of 10^{-8}. The total power available to the system for

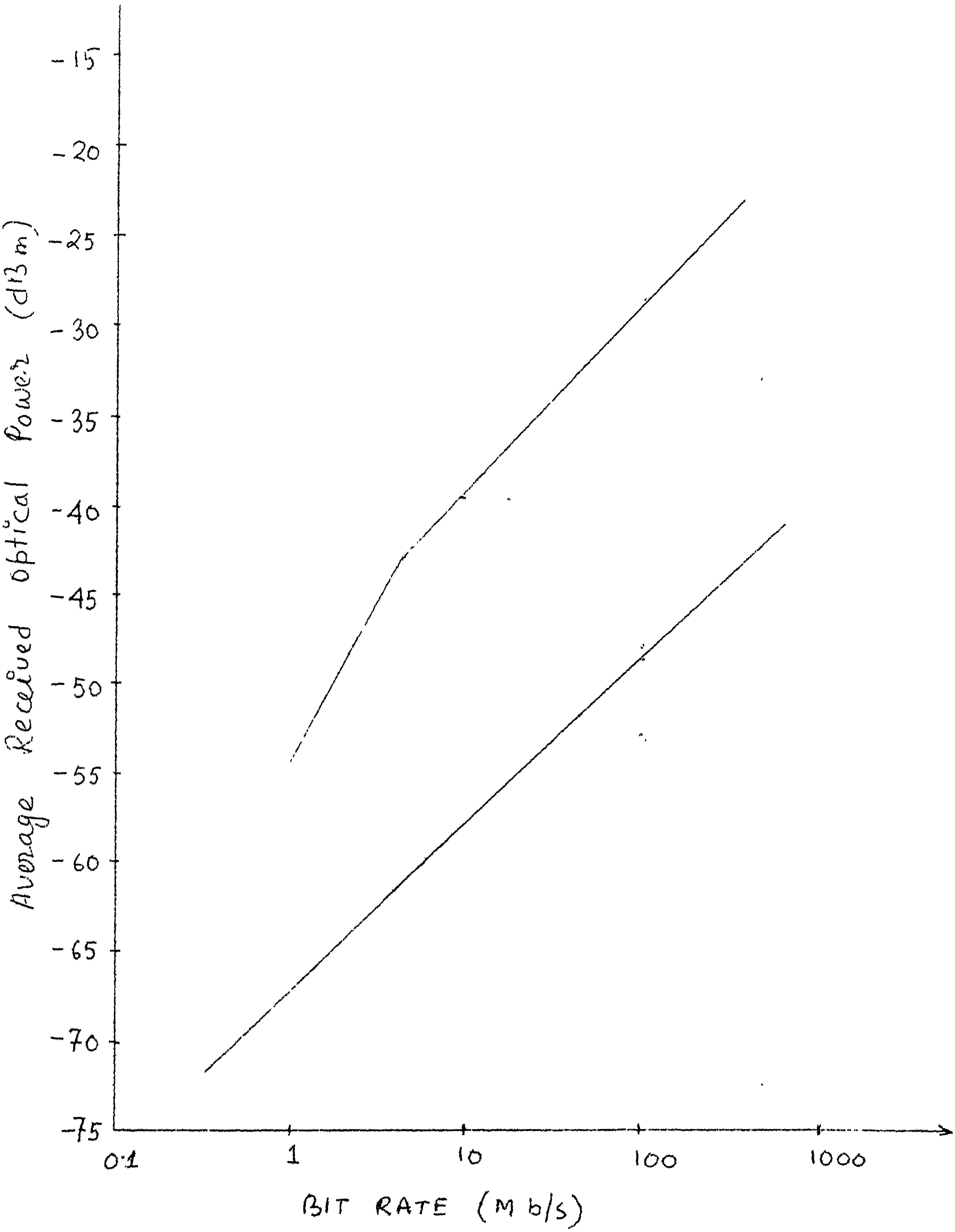


FIG 3·2 Required optical power Vs Bit rate for digital syste

PIN detector under the condition of Fig. 3.2 may be described as F(T), and is given approximately as

$$F(T) = -110 + 4.35\, \ell_n \,(T^{-1}) \qquad (3.18)$$

where 'T' is the bit interval.

This power is shared in various ways in the operating system and an operating margin must be maintained for the electronics so that small fluctuations do not effect the performance of the system. Typically a margin of 5 dB is used. The fibre itself is characterized by a loss in dB/km, and in addition, there will be power loss due to joints, mode coupling, etc.

Due to the dispersion effect the power transmitted in a given bit interval will partly arrive in the corresponding bit interval at the receiver, and the rest of the power will spill over into the nearest neighbour interval or even to more intervals depending upon the impulse response of the fibre. This overlap between the adjacent pulses, causes what has been referred to as the ISI. Thus the overall system can not be designed unless ISI is taken into account. The ISI at a finite data rate in the system could be reduced by equalization in the receiver after the detector, but only at a penalty it pays. The penalty is described by F_{ISI} and measured in decibels. Therefore,

for a given bit rate, the design equation becomes

$$F(T) = 110-4.35\, l_n\,(T^{-1})+P_L = \alpha(L) + M + F_{ISI} \tag{3.19}$$

where P_L is launched power (dBm). We will use eqn. (3.19) in the next section for link calculation.

3.4 FIBRE OPTIC DIGITAL LINK CALCULATIONS:

The Performance of the digital fibre optic link depends upon many factors, viz., source, length of fibre, attenuation constant of fibre, photodetector and coupling loss. As already explained, fibres are dispersive in nature, thus pulse propagated through the fibre will get spread in time and cause ISI. This will limit the data rate that can be transmitted over a given fibre optic link.

As discussed in Chapter 1, we have used a PIN detector for the system under study. The link calculation for the fibre optic communication system under study for different lengths and sources are given in Table 3.1. Specifications of the components used for link calculation are as follows:

a) Source:

i) Laser Diode

Output power : mW (0 dBm)

Table 3.1: Link Calculation for the F.O.C. Digital System

A. Source-Laser Diode

i) Fibre length : 2.155 km

ii) Fibre length : 4.310 km

Source Power : 1 mW (0 dBm)

Connector Insertion loss : 2 dB

Total Power available to system:

$$F(T) = -110 + 4.35 \ln (T^{-1})$$

Data Rate in M bits/sec	100	150	200	250	300
Total power	29.87	28.10	26.85	25.88	25.09
Connector Insertion loss	2.0	2.0	2.0	2.0	2.0
Operating margin	5.0	5.0	5.0	5.0	5.0
Power available to transmission media	22.87	21.10	19.85	18.88	18.09
Fibre loss for 2.155 km (4.5 dB/km)	9.69	9.69	9.69	9.69	9.69
Power available for ISI penalty (dB)	13.18	11.41	10.16	9.19	8.40
Fibre loss for 4.310 km (4.5 dB/km)	19.39	19.39	19.39	19.39	19.39
Power available for ISI penalty (dB)	3.48	1.71	0.46	-	-

B. Source-LED

i) Fibre length : 2.155 km

Source Power : 50 μW(-13.01 dBm)

Total power available to system : $F(T) = -110 + 4.35 \ell_n(T^{-1})$

Date Rates in M bits/sec.	75	100	125
Total power	31.12	29.87	28.89
Coupled power to system	13.01	13.01	13.01
Connector insertion loss	2.0	2.0	2.0
Operating margin	5.0	5.0	5.0
Power available to transmission media	11.11	9.86	8.88
Fibre loss for 2.155 km	9.69	9.69	9.69
Power available for ISI penalty (dB)	1.42	0.17	-

ii) LED - IRE - 161 (LASER-DIODE)

Output Power : 50 μw at NA = .2

Current : 100 mA

Core : 50 μm

b) Fibre - (MG -05):

Length : 2.155 km

Attenuation: 4.5 dB/km

NA : 0.23

Core : 63 μm

Material dispersion : 90 p.sec/km-nm

c) Connector OFP-101

Insertion loss = 1 dB

d) Detector

PIN Diode - OFDP-04 (ITT, UK)

Dark current : 0.2 nA

Sensitivity : As shown in Fig. 3.2 for BER 10^{-8}

Responsivity : 0.5 A/w

It is assumed that the connector insertion loss is same for the laser diode and the LED. Also connector loss for the two sections of fibre is assumed to be negligible.

From the link calculations we observe that with the laser diode the system can work upto a data rate of 200 M-bit, for a fibre length of 4.310 km whereas with the LED the system can work only upto a data rate of 100 M-bits for a fibre length of 2.155 km. As for data rates higher than this there is no power available for meeting the ISI penalty. Therefore, to achieve the given BER of 10^{-8} at higher data rates, either one should use the laser diode or decrease the length of the fibre. Link calculations with a laser source for a fibre length of 2.155 km varifies the latter suggestion.

3.5 IMPULSE RESPONSE:

Impulse responses of the various parts of the fibre optical system and the overall link have been given in the following sections.

3.5.1 General:

Impulse response of an optical fibre is generally found by measurements of the delay distortion. In the system under discussion a graded index fibre is used, and the measured impulse response of the fibre provided by the manufacturer is shown in Fig. 3.3. It is assumed that the impulse response is normalized and the time is given in nano-secs. The suppliers have not given the exact value

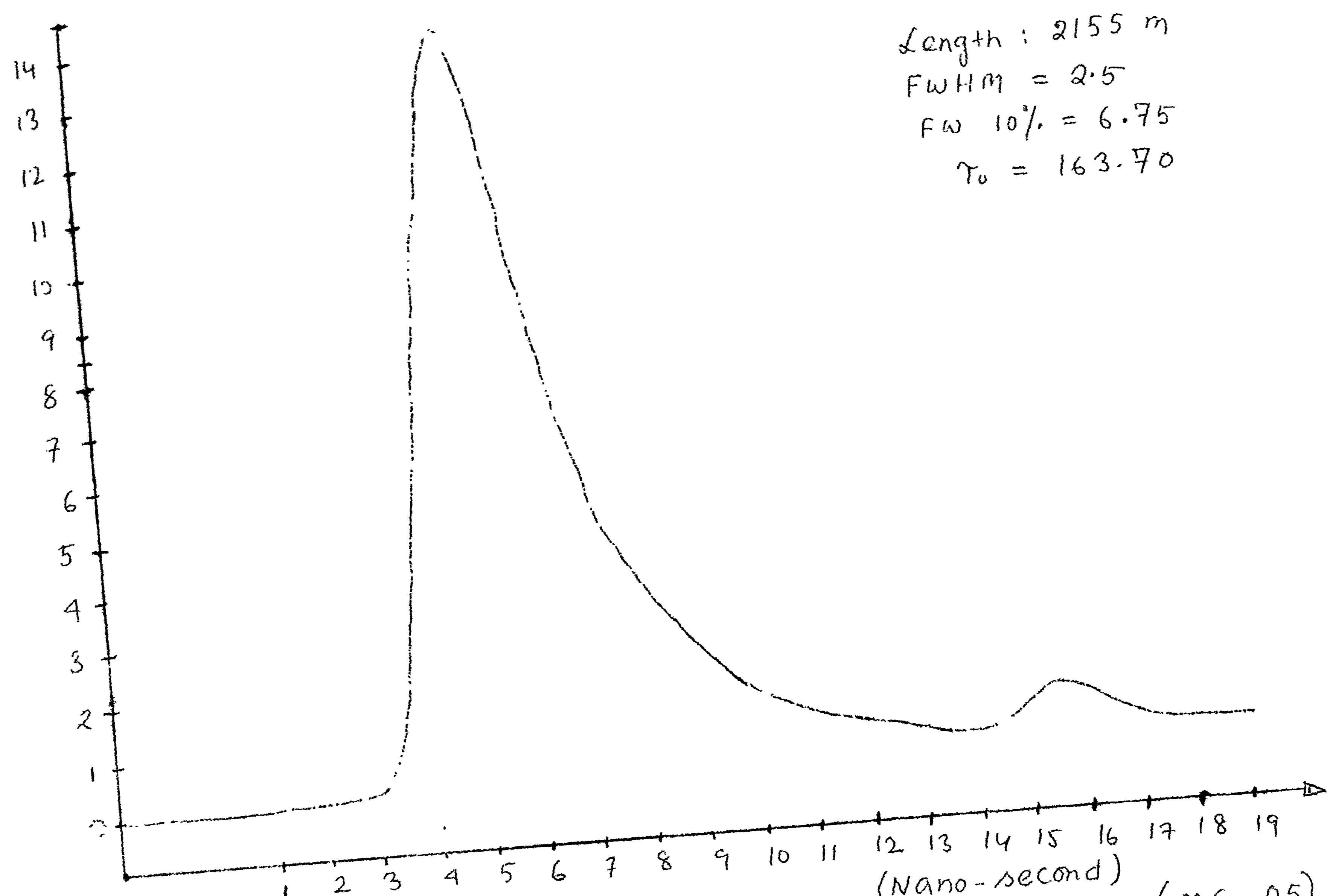


FIG 3.3 IMPULSE RESPONSE OF FIBER (MG-05)

of the magnitude (scaling factor). The value of the scaling factor is found as follows:

We know –

$$\int_0^{\infty} (Kh)^2 t \;\; dt = 1 \quad \Delta_t \quad \text{-- small time interval}$$

or,

$$K^2 \;\; \Sigma \; h^2 i \, \Delta_t \;\; = 1 \qquad (3.20)$$

For calculating K, Δ_t is taken as 0.1 ns. and the value from calculation turns out to be 1.55×10^3. As we have taken only a finite number of samples of the impulse response the exact value of K is 1.5×10^3.

3.5.2 Impulse Response of the Electrical System:

The optical signals are converted to electrical signals by the PIN diode detector and then these are amplified by the pre-amplifier. As the data rates of interest are high, a high speed FET (CA 3127E) is used for the pre-amplifier circuit. The characteristic of CA3127 and the PIN diode are as under –

CA3127E

Input resistance = 400 Ω

Input capacitance = 3.7 pF

PIN Diode

Capacitance = 1 pF

The equivalent electrical circuit of the PIN diode and the pre-amplifier is given in Fig. 3.4.

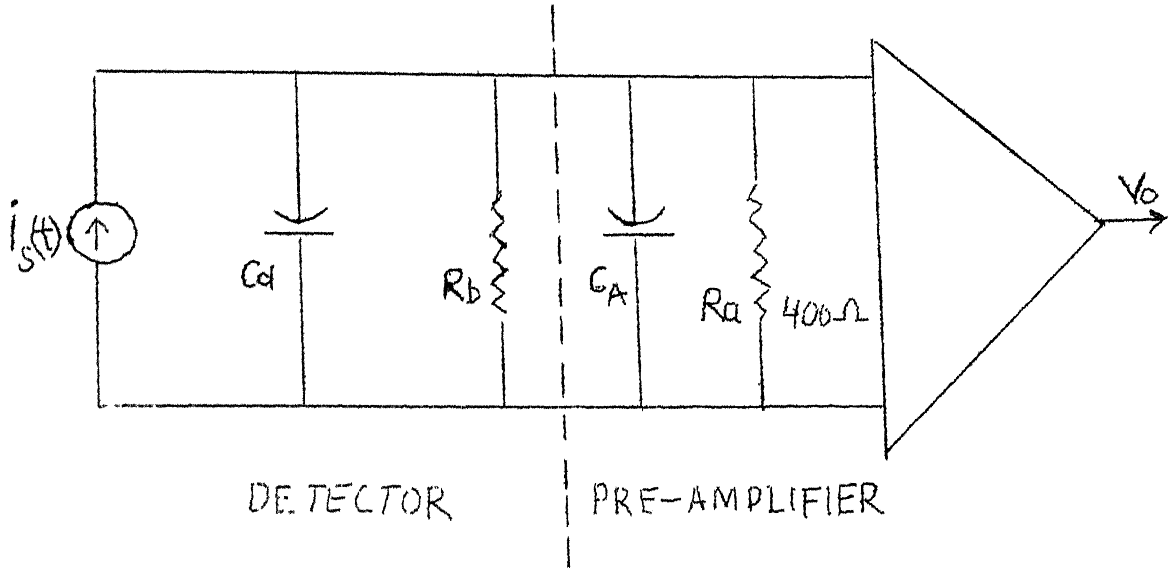


Fig.3.4

Current due to the shot noise and dark current are not considered being negligible. The bias resistance of the photodetector is high and shunted by the 400 Ω input resistance of the pre-amplifier. The resultant values of R and C of the model are

$$R = 400\ \Omega$$

$$C = 4.7\ \text{pf}$$

The impulse response of the detector and the pre-amplifier is then

$$H(s) = \frac{V_2(s)}{V_1(s)} = \frac{R \cdot \frac{1}{C_s}}{R + \frac{1}{C_s}} = \frac{\frac{1}{C}}{S + 1/R_C} \qquad (3.21)$$

$$\therefore h(t) = 1/C \cdot e^{-1/R_C t} \qquad (3.22)$$

Impulse response of the equivalent circuit is plotted as shown in Fig. 3.5.

3.5.3 Impulse Response of the System:

To evaluate the performance of the DFE, the total impulse response of the system is considered. The fibre optical communication links under consideration are as follows:

a) Source-Laser Diode, Detector-PIN Diode Link length = 2.155 km

b) Source-LED, Detector-PIN Diode, link length = 2.155 km

c) Source-Laser Diode, Detector-PIN Diode Link length = 4.310 km

a) <u>Laser-PIN Combination, Length = 2.155 km:</u>

The material dispersion of the graded index fibres is 90 p.secs/km.-nm. It is assumed that the spectral width of the laser is 2 nm. Thus the total material dispersion due to the laser is very small, i.e., 0.45 nsecs. Therefore, the effect of material dispersion due to the

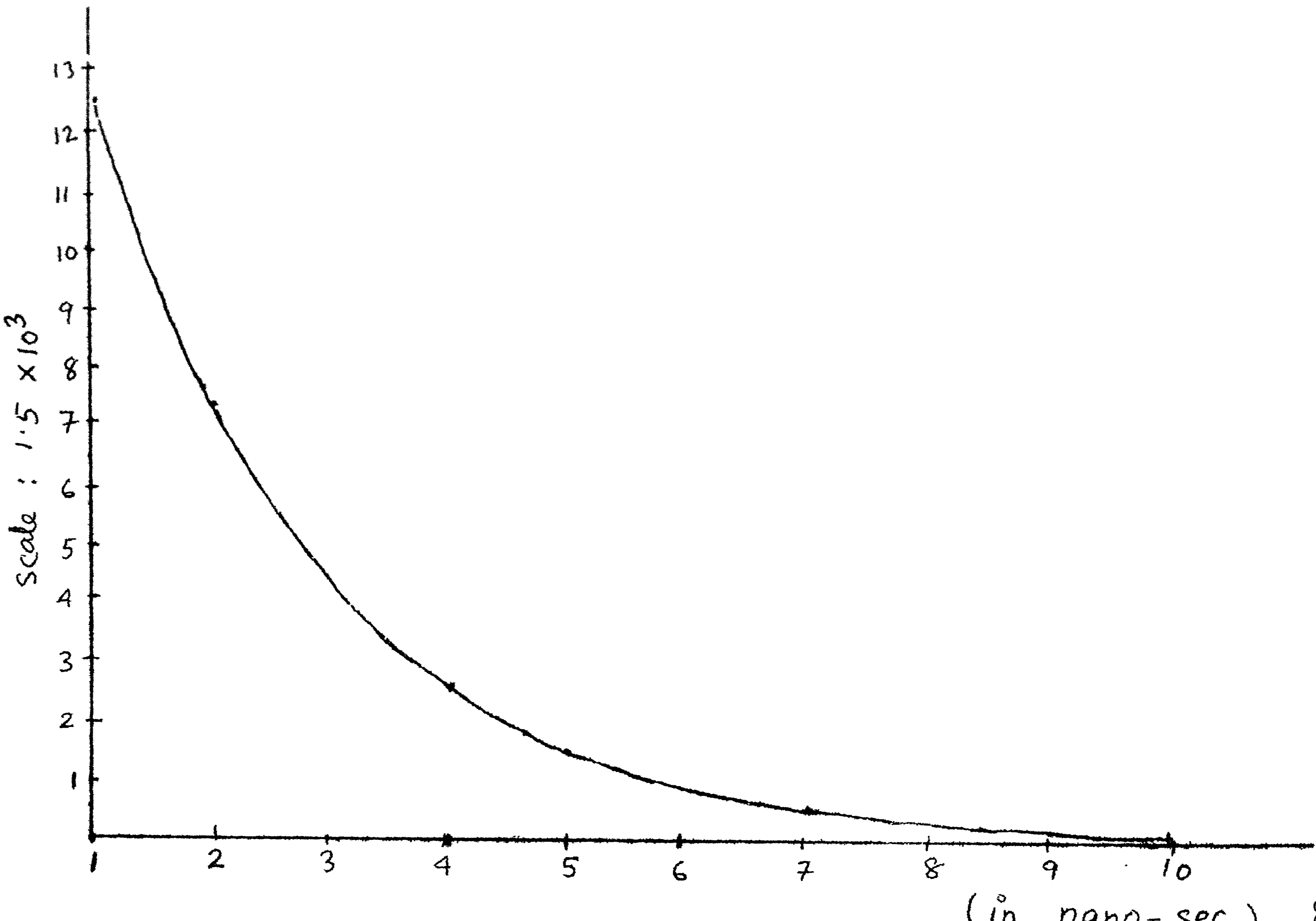


FIG 3·5 IMPULSE RESPONSE OF ELECTRICAL CIRCUIT

source in this case is neglected while calculating the total impulse response of the system. We know the impulse response of the fibre and the electrical circuit (detector + pre-amplifier). The total impulse response is found by convolving the two impulse responses. The total normalized impulse response is shown in Fig. 3.6.

b) LED-PIN Combination Length = 2.155 km:

Optical fibre properties depend upon the wavelength λ of a light source, i.e., $g(t) = g(t, \lambda)$, where $g(t)$ is the impulse response of the fibre. The spectral width of the LED is 40 nm. Therefore, the material dispersion will be much more compared to the laser source. The LED, IRE-161 of Laser Diode Lab. is used in the fibre optic communication link under discussion. The output spectrum of the light source is as shown in Fig. 3.7.

The resultant pulse response of the fibre link is [22]

$$g_r(t) = \int_{-\infty}^{\infty} p(\lambda) g(t, \lambda) d\lambda \tag{3.23}$$

where $p(\lambda)$ is high source spectrum.

The resultant impulse response is shown in Fig. 3.8. If $p(\lambda)$ is not available for a LED, we can use equation (3.24) where a Gaussian intensity spectrum for the LED has been assumed.

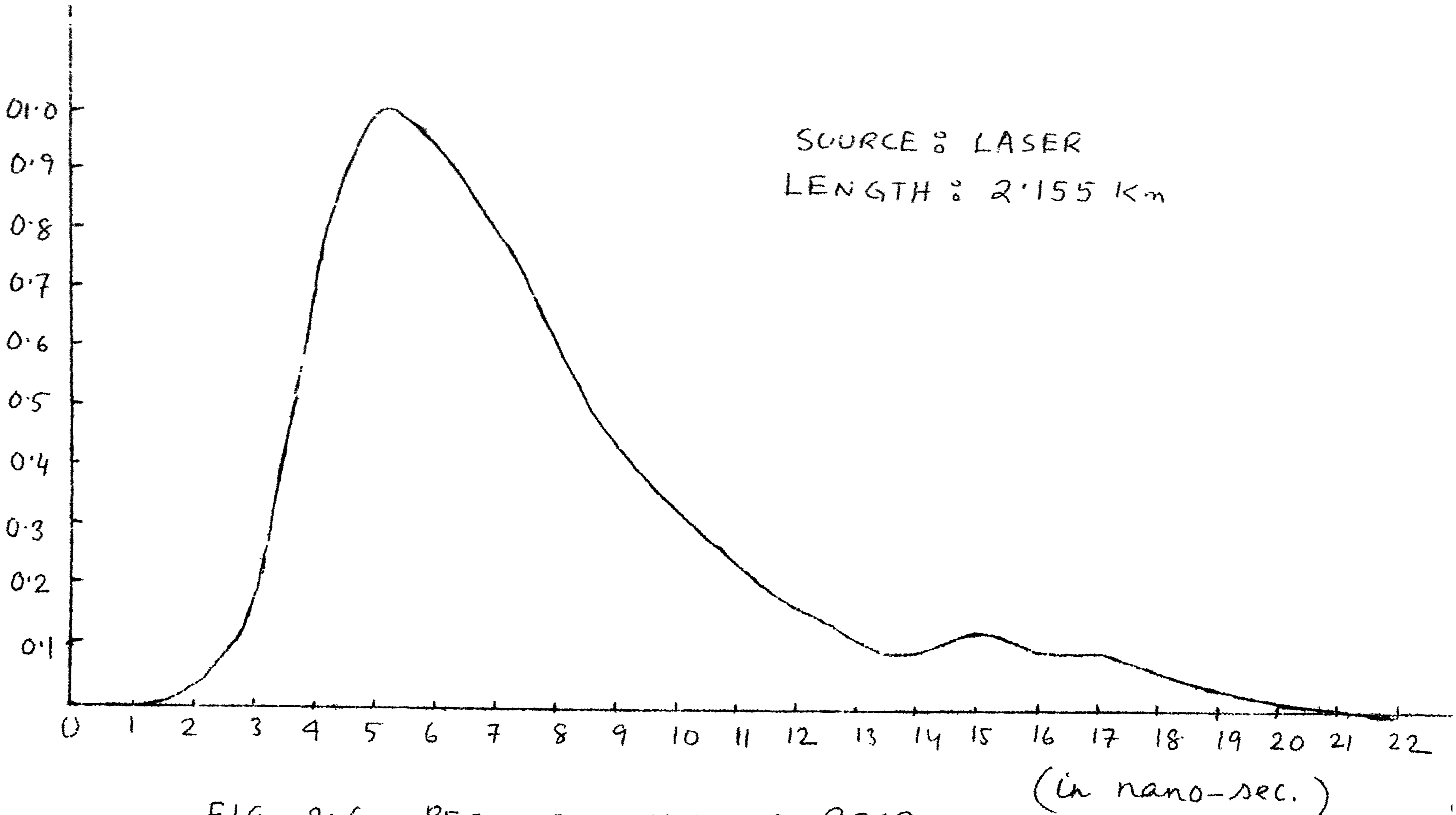


FIG 3·6 RESULTANT IMPULSE RESPONSE

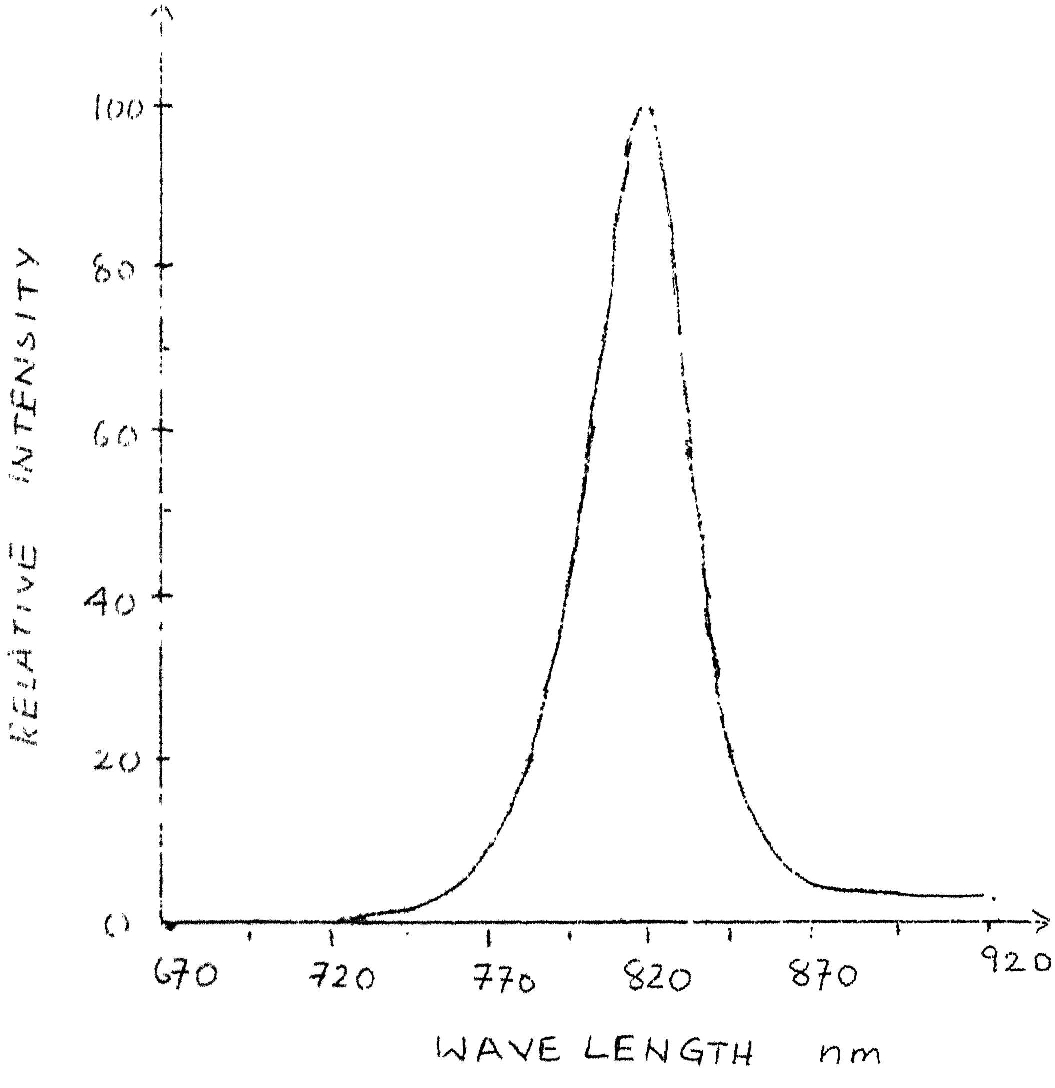


FIG 3·7 OUTPUT SPECTRA

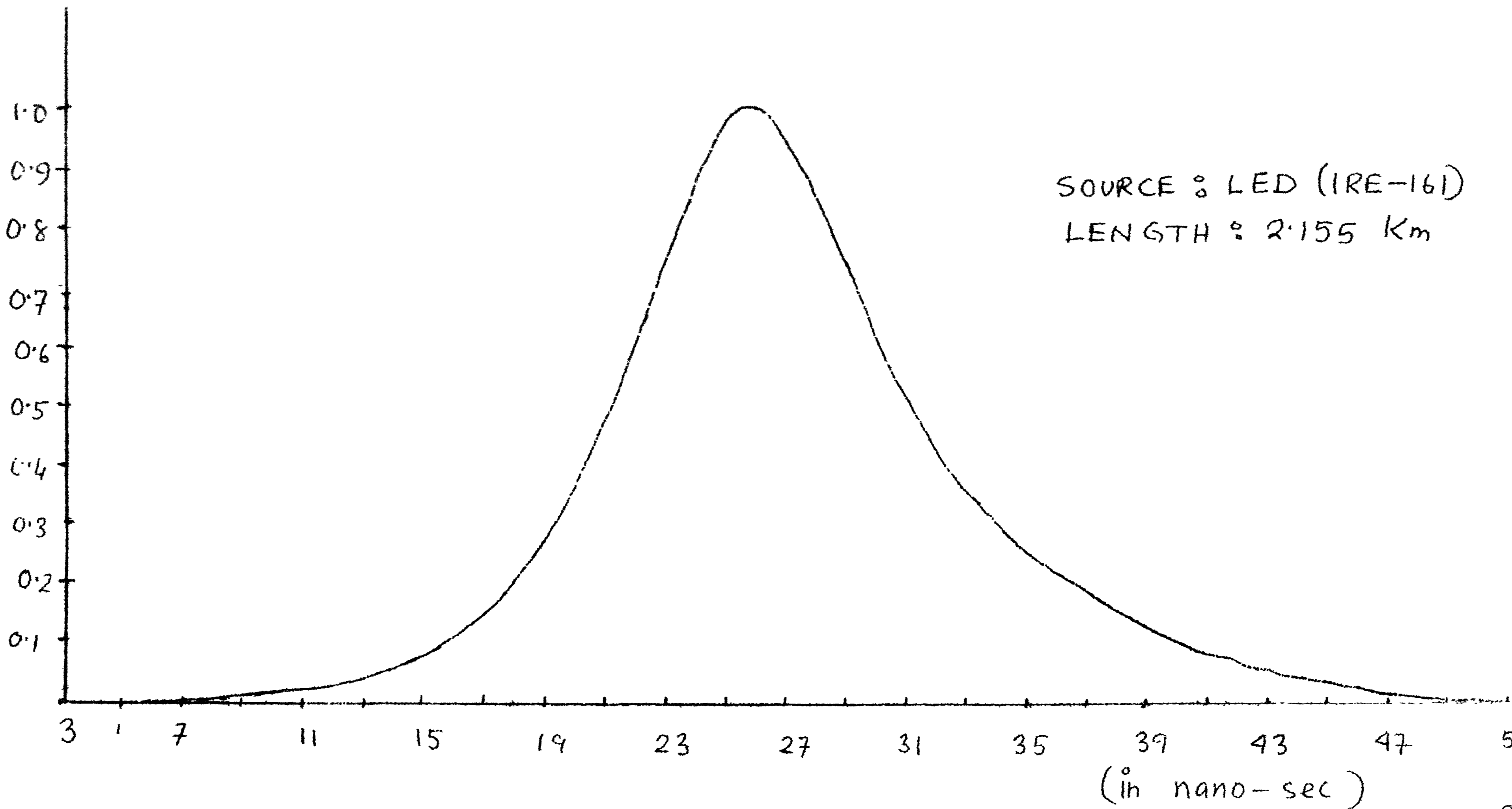


FIG 3·8 RESULTANT IMPULSE RESPONSE

$$G_m(f) = \exp\left\{-\frac{1}{2}\cdot(2\pi fML\sigma)^2\right\}$$

or,

$$g_m(t) = \frac{1}{\sqrt{2\pi}ML\sigma} \exp\left\{-\frac{1}{2}(t/ML\sigma)^2\right\} \qquad (3.24)$$

where M = material dispersion in ps/km-nm

L = length of fibre in km

σ = spectral width of LED in nm

The overall response will then be obtained by convolving $g_m(t)$ with $g(t)$. The output spectra of the light source IRE - 161 is not Gaussian. The value of σ is found by curve fitting. It is 14 n.secs on the right hand side of the peak value and 17 n.sec. on the left hand side. In our case the resultant impulse response of the link is obtained numerically by using the actual $p(\lambda)$ as provided by the manufacturer.

The plots of Fig. 3.9 and 3.10 have been obtained using the Gaussian assumption and using values of σ of 14 and 17, respectively. The resultant impulse response shown in Fig. 3.8 corresponds to a worst case situation.

c) Laser-PIN Combination, Length = 4.310 km:

The total impulse response of the system is found by convolving the impulse response of the fibre of 2.155 km

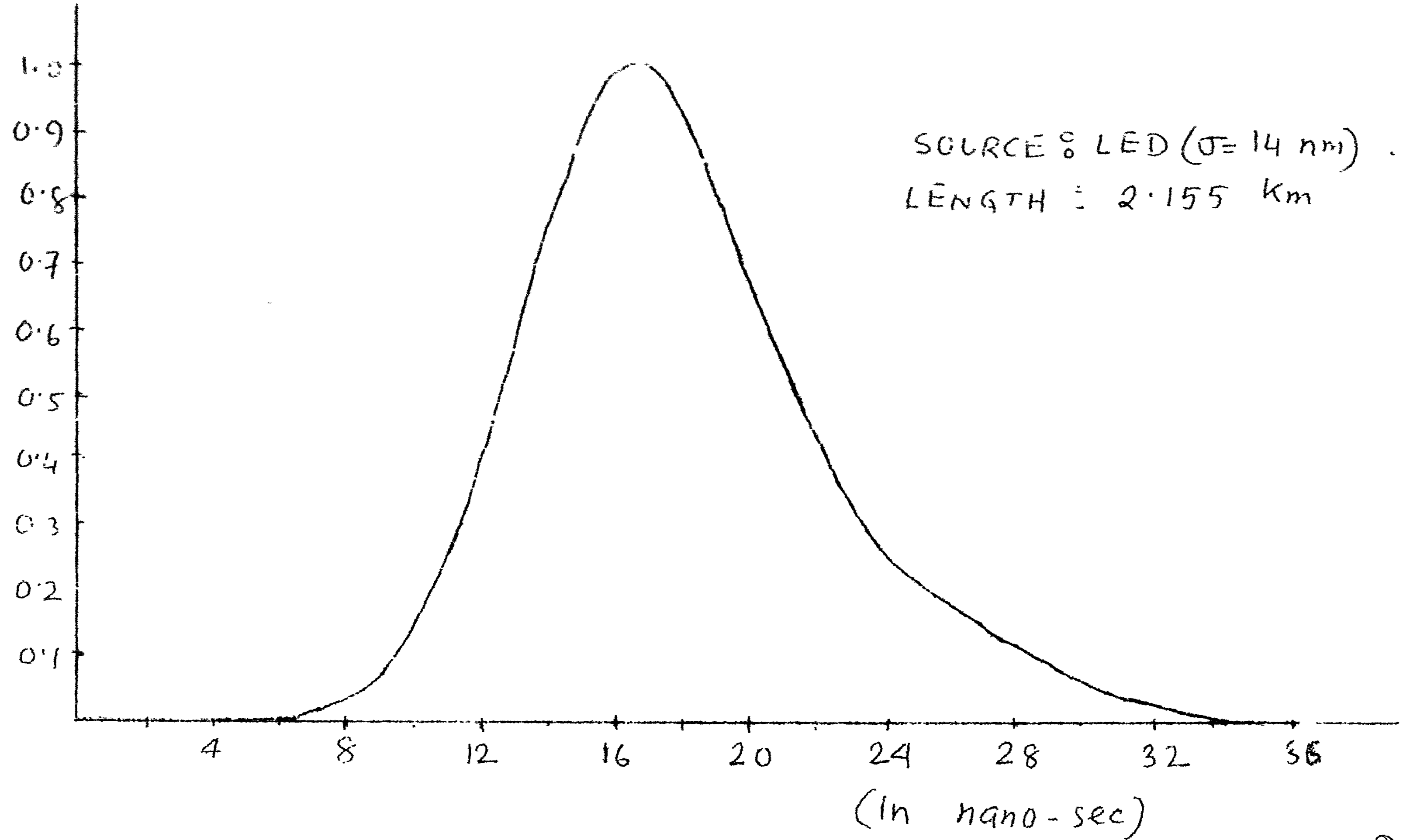


FIG 3.9 RESULTANT IMPULSE RESPONSE

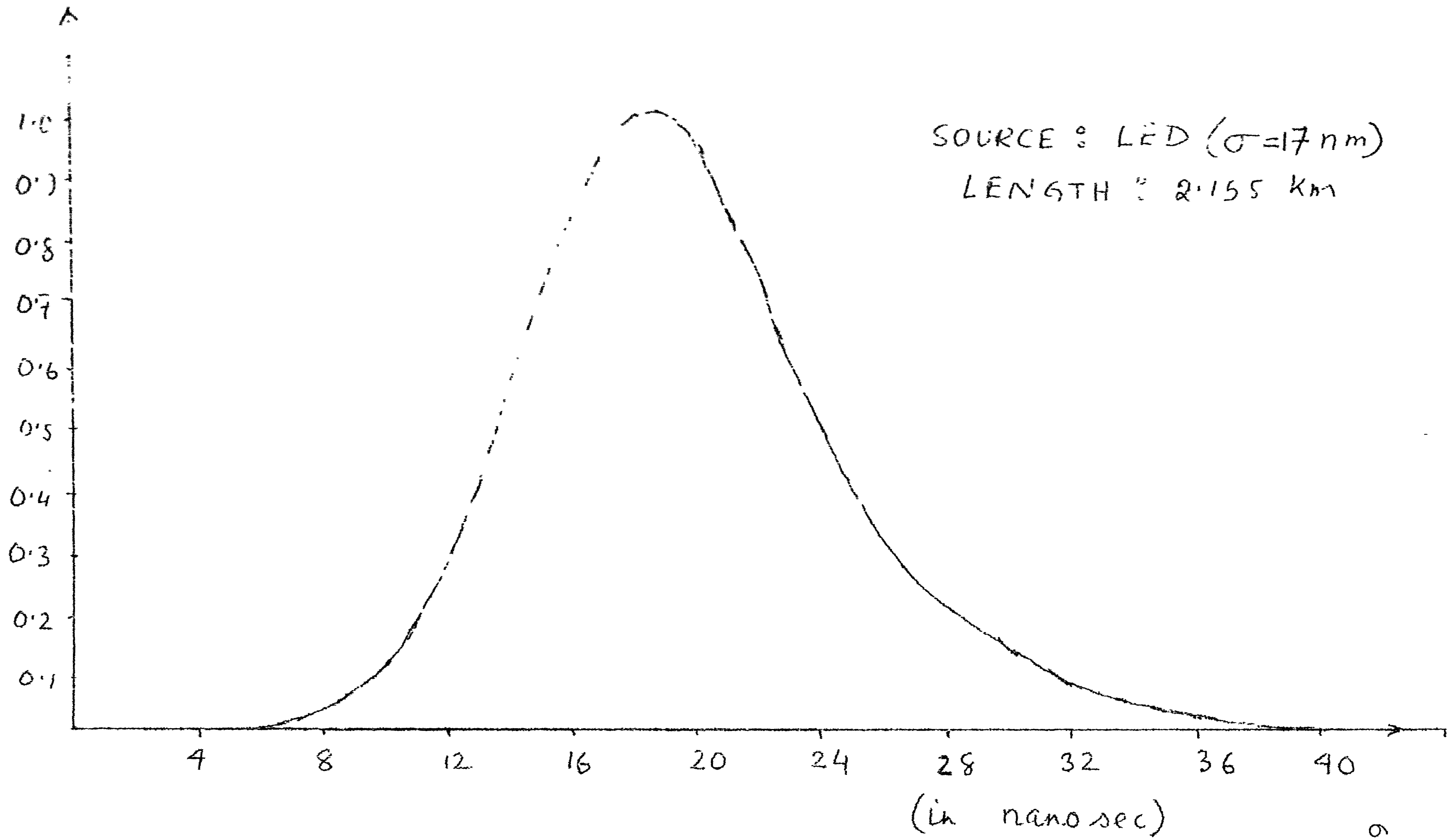


FIG 3.10 RESULTANT IMPULSE RESPONSE

length with itself and then with the electrical circuit impulse response. Total impulse response is shown in Fig. 3.11 which is used to find the probability of error of the system of fibre length 4.310 km.

3.6 CHANNEL BAND-WIDTH OF THE SYSTEM:

As discussed in Chapter 1, the system can be divided into two parts, i.e. optical and electrical.

The system under discussion have electrical as well as optical portions. The 3 dB channel bandwidth of the combined system is calculated.

It is observed from Section 3.2 if the channel is band limited to B/2, it is reasonable to position a filter prior to the equalizer which will also band limit the additive noise to the range $|f| \leq B/2$. Therefore one should know the channel bandwidth of the system upto the input to the equalizer.

We have the pre-amplifier before the equalizer, so the additive noise is also limited to the bandwidth of the pre-amplifier. Therefore, an additional filter to band limit the additive noise is not required.

The impulse response of the fibre or the overall system in our case is known. For the impulse response $h(t)$, $0 \leq t \leq \tau$, the transfer function is

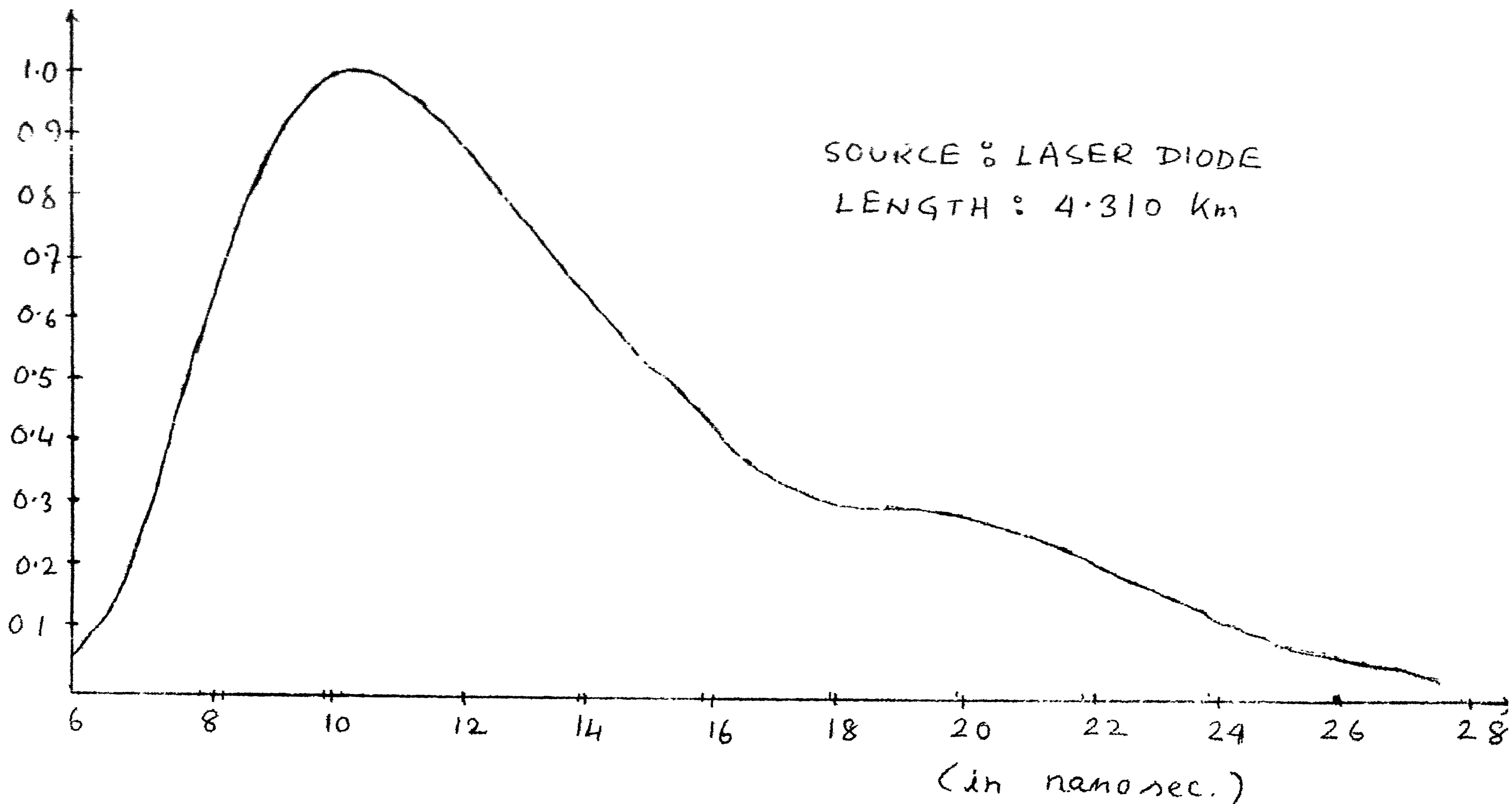


FIG 3.11 RESULTANT IMPULSE RESPONSE

$$H(f) = \int_0^{\Upsilon} h(t) \cos 2\pi f t dt + j \int_0^{\Upsilon} h(t) \sin 2\pi f t dt$$

$$= A(f) + jB(f)$$

$$|H(f)|^2 = A(f)^2 + B(f)^2$$

where $A(f) = \int_0^{\Upsilon} h(t) \cos 2\pi f t dt$ and

$$B(f) = \int_0^{\Upsilon} h(t) \sin 2\pi f t dt$$

The integral functions are computed numerically. The 3 dB channel bandwidth is obtained from the plot of $|H(f)|^2$.

The magnitude of the transfer functions of the electrical circuit, the fibre optics, and the total system (a) are plotted in Fig. 3.12 and Fig. 3.13. The values of the 3 dB bandwidths are obtained as under:

i) Electrical Circuit = 85 MHz
   Fibre optic = 45 MHz
   Total bandwidth of system (a) = 38 MHz
   Total bandwidth of system (b) = 24 MHz

3.7 DESIGN OF DFE:

In the following the choice of the step size (α) and the tap gain coefficients of the DFE have been dealt with in brief.

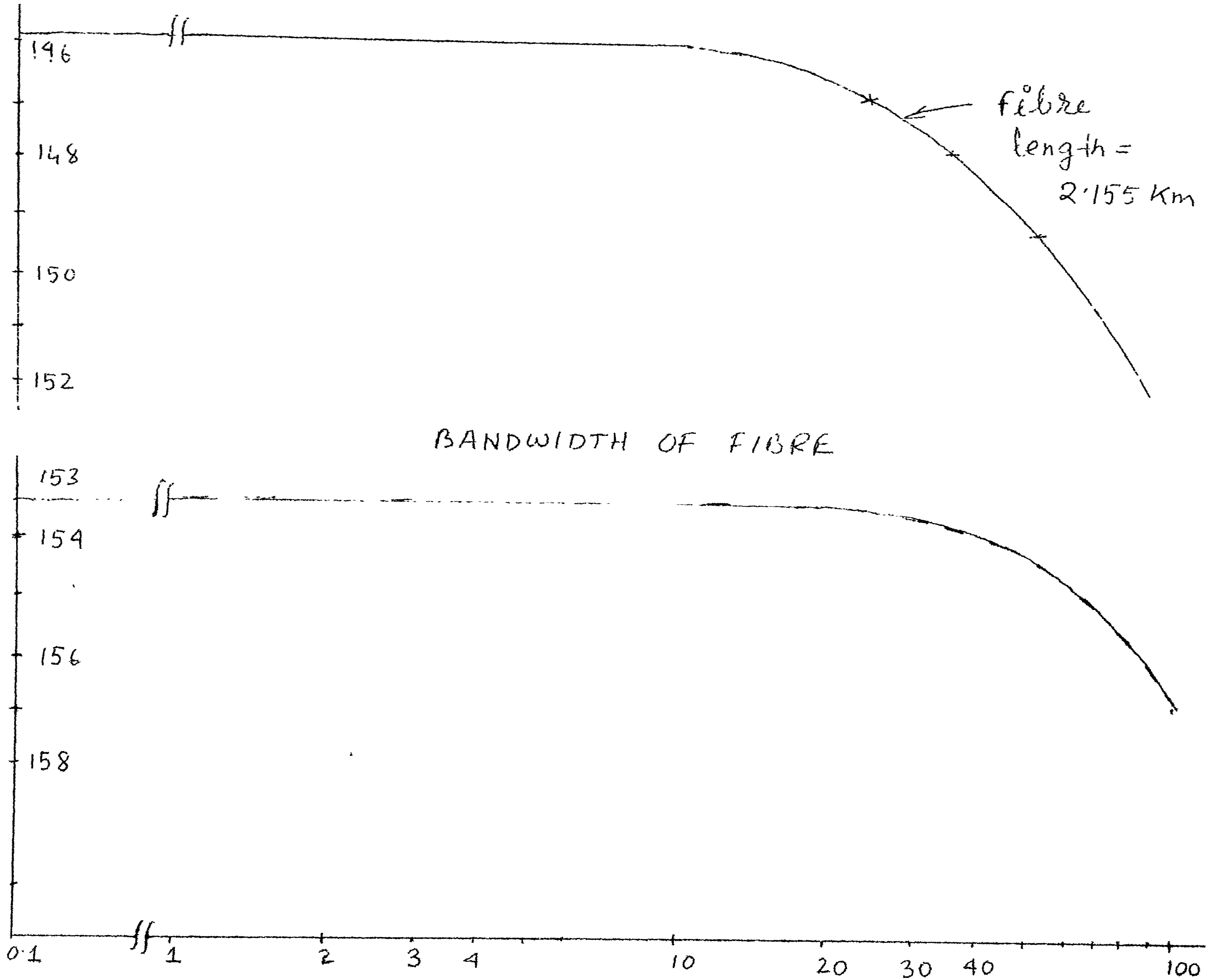


FIG 3:12 BANDWIDTH OF PRE-AMPLIFIER & DETECTOR

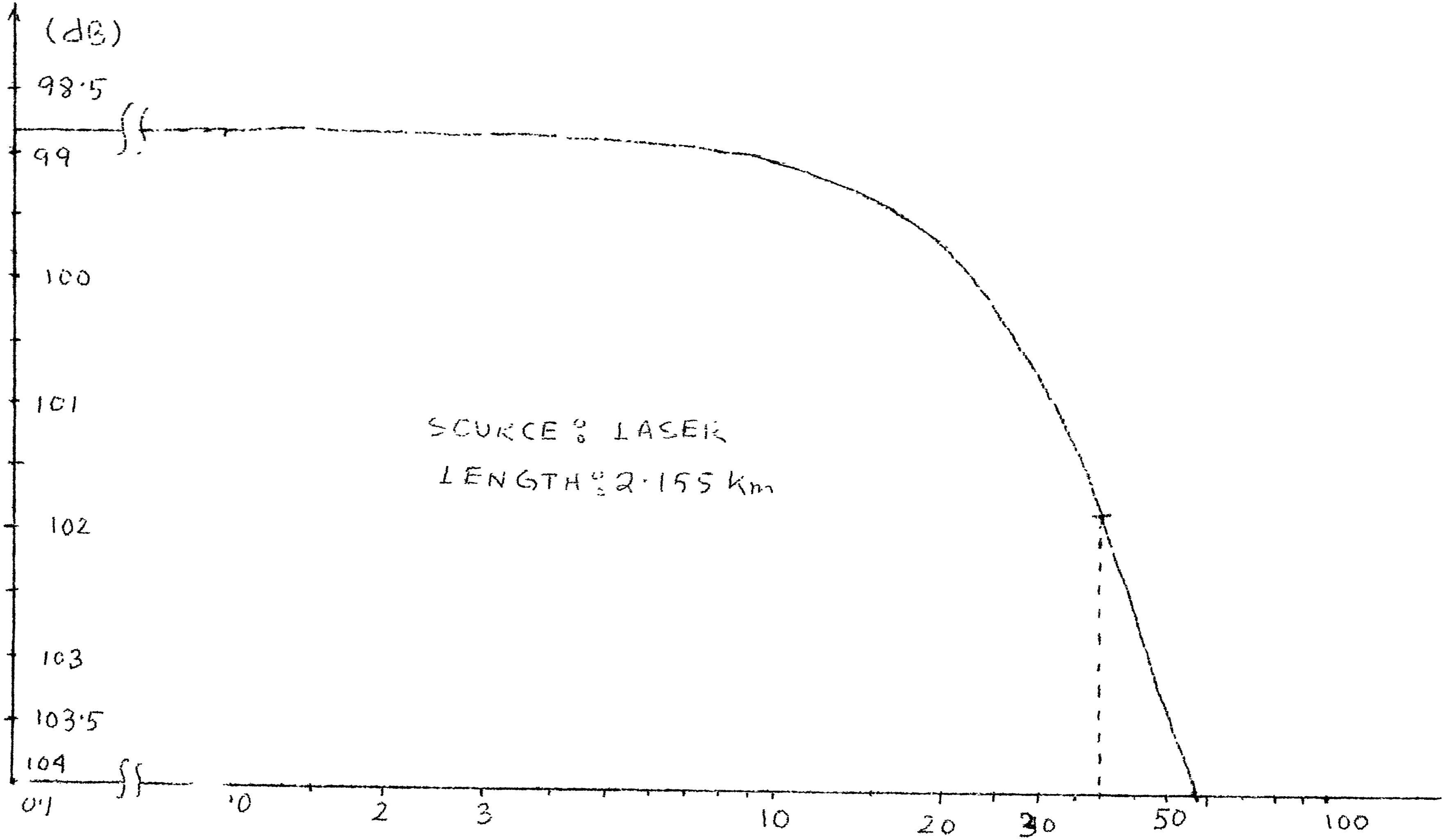


FIG 3 13 BANDWIDTH OF SYSTEM

3.7.1 Effect of Step Size (α):

The critical consideration in the choice of the algorithm parameters is to maintain a misadjustment small relative to unity.

The algorithm parameters which can be varied are the step size α and the averaging interval. We have already assumed the averaging interval to be unity. α should make the receiver tap gains move to the optimum neighbourhood from an initial setting corresponding to the absence of an equalizer. Thus the problem is to maximise the convergence rate.

While carrying out simulation on the digital computer, it is observed that if we make α larger then the probability of error is more, though convergence is faster, on the other hand if the step size is kept too small then the convergence rate is very slow. The algorithm will converge if α is chosen so that

$$\rho(\overline{\underline{V}\,\underline{V}'}) < \frac{2}{\alpha}$$

D.A. George [23] has used a value of α as low as 0.016.

In this thesis we assumed that the reconstructed data sequence has no error. Therefore, α should be so chosen that the probability of error is low. When the equalizers

tap gains can adaptively move to the optimum neighbourhood from the initial setting, the equalizer is said to have achieved lock.

In the simulation the tap gains of the forward and the backward filters are initially set to zero except for the tap corresponding to F_0 (main tap in the forward transversal filter), which is set to unity. It is noticed during the simulation that if after a number of iterations the weight of F_0 is around unity, then the best tap gain settings are achieved. But in case the value of F_0 tap converges to zero then lock is not achieved. The error rate becomes very high after a few iterations. The forward filter tap weights set to zero and backward filter tap weights set to such a value that their sum tends to be 1 or -1. In this case the reconstructed data is independent but the error, $e_K = \widetilde{S}_K - \hat{S}_K$ is identically zero for all times. Therefore, the selection of α is very important and should be chosen so as to achieve the lock condition. Also the tap weights of F_0 should not converge to zero. The value of F_0 should be around unity to get the best results.

3.7.2 Tap Gain Setting:

<u>Length of transversal filter</u>: The parameters of interes in the DFE are the number of taps in the forward filter, i.e.

$-N$ to N, the forward filter tap spacing τ and the number of taps in the backward filter, say M. The optimum DFE requires $N = \infty = M$ and $\tau = \frac{1}{B}$, where $1/B$ is the Nyquist interval. In practice the transversal filter will be of finite length.

Monsen [24] in his studies kept tap spacing for the forward filter as 66 ns (corresponding to 15 MHz), data rate between 1.5 to 12.6 M bits/sec. and three taps in the forward filters. In the system under study the channel bandwidth is about 38 MHz, and data rates much higher than the channel bandwidths are used in simulation. From the simulation studies, the following observations can be made about the number of taps and their spacing.

i) When the forward filter taps are spaced at the symbol interval, the probability of error is less than or similar to the value obtained when the taps are spaced at the Nyquist interval.

ii) If we keep the number of the forward taps equal to the number of future interfering samples plus one then the probability of error is smaller.

iii) Any additional tap added in the forward filter after the main tap F_0 does not reduce the probability of error. The results are tabulated in Table 3.2.

Table 3.2

(a) Taps at Symbol Interval

Data Rate in M Bits/sec. (a)	SNR (b)	No. of symbol Processed (c)	Step size (α) (d)	Normal setting ($F_o=1$) No. of taps Forward (e)	Normal setting No. of taps Feedback (f)	Normal setting Error (g)	Different setting No. of taps Forward (h)	Different setting No. of taps Feedback (j)	Different setting Error (k)	Remark (l)
150	6	1000	.01	1	3	22	2	3	28	$F_o=1$
150	6	1000	.01	1	3	22	3	3	30	$F_o=1$
225	6	2000	.01	2	4	60	5	4	68	$F_2=1$
150	8	2000	.01	1	3	10	3	3	13	$F_1=1$
150	8	10000	.1	1	3	91	3	3	93	$F_1=1$
225	6	2000	.01	2	4	60	4	4	69	$F_3=1$(two taps after mai
150	8	2000	.1	1	3	10	3	3	17	$F_1=1$(one tap before
150	8	2000	.01	1	3	11	5	4	14	$F_3=1$ one tap a
225	6	2000	.01	2	4	60	3	4	72	$F_1=1$ main ta
(b) Taps at Symbol and Nyquist interval										
150	6	1000	.01	1	3	22	2	3	26	Forward tap (Nyquist inter
150	8	2000	.01	1	3	10	5	3	12	$F_2=1$
150	8	10000	.1	1	3	91	5	3	103	$F_2=1$
150	4	1000	.01	1	3	39	2	3	43	$F_o=1$
225	6	2000	.01	2	4	60	2	4	65	

CHAPTER 4

SIMULATION OF DFE AND RESULTS

4.1 GENERAL:

To evaluate the performance of DFE, the actual system is simulated on the DEC-1090 system. The overall impulse responses of the link are considered for the simulation of the system. The simulation consists of finding the tap gain weights for the forward and backward transversal filters and then using these tap gain weights to find the probability of error at different data rates for various SNR values. The probability of error is determined for 2.155 km and 4.310 km link lengths for SNR values from 6 dB to 14 dB. Performance of the DFE for these link lengths is evaluated considering the different optical sources, i.e., laser - diode and LED. Maximum data rates possible for a certain probability of error without an equalizer have also been found.

For simulation random numbers with uniform distribution (data) and Gaussian distribution with zero mean (noise) are generated by NAG sub-routine. The uniformly distributed random numbers are convolved with the total impulse response of the system, and the Gaussian noise after convolving with the pre-amplifier and detector impulse response is added to it. Now the signal is fed to the DFE.

Signal flow diagram for the simulation is shown in Fig. 4.1. Error signal e_K is used to update the tap weights during the adaptation process. Once the tap weights of the DFE are fixed, error signal e_K is not used.

4.2 SIMULATION RESULTS:

4.2.1 Source-Laser, Fiber Length = 2.155 km:

The simulation is carried out in two parts. First we find the optimum tap weights for the forward and backward filters for the different data rates at various SNR values by the adaptive method. As discussed in Chapter 3, the forward filter having number of taps equal to the future interference samples plus one and spaced at the symbol interval is considered in evaluating the optimum tap weights and the probability of error. For various values of SNR the noise variance and the standard deviation is given in Table 4.1.

The optimum tap weights are determined by carrying out a number of simulations with different values of step size and then freezing the values of the tap weights at different iteration numbers. The tap weights which correspond to the minimum number of errors in the above simulation is used to find the probability of error of the system. The results from one such simulation are given in Table 4.2.

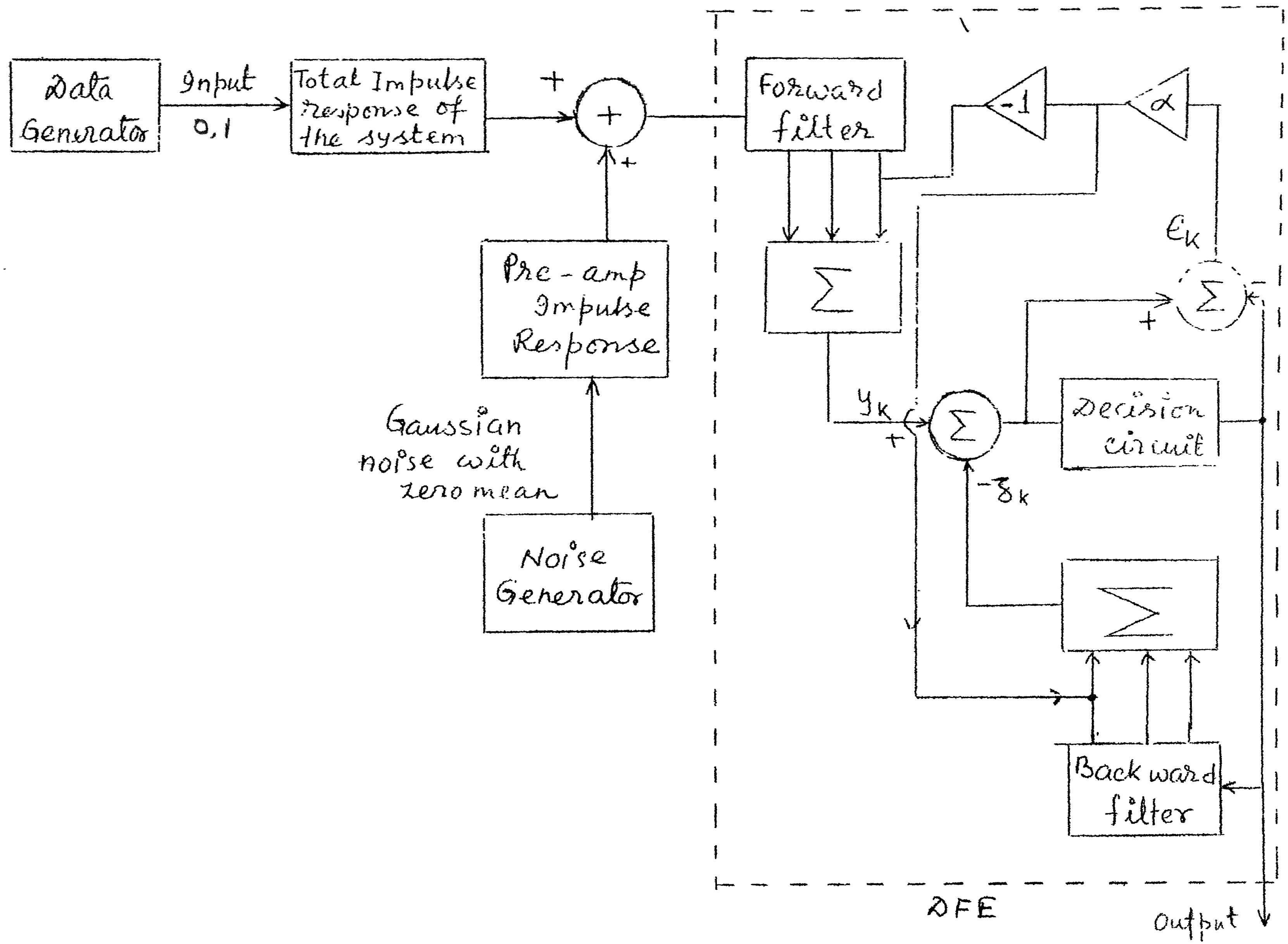


FIG 4.1 SIGNAL FLOW DIAGRAM

Table 4.1: Noise Variance and Standard Deviation at Different SNR Values

SNR	Noise Variance	Standard Deviation
6	0.2512	0.5012
8	0.1585	0.3981
10	0.1	0.316227
12	0.0631	0.251188
14	0.0398	0.199526
16	0.02512	0.15849
24	0.00398	0.0631
32	0.00063	0.02512

Table 4.2

SNR = 10 No. of sample processed = 5000

Data Rate 150 M bit/sec.

Step size	Error	Freezed value at	Tap weight Forward (1)	Backward (1)	(2)	(3)
0.10	7	495	0.9803	0.08568	-0.06226	0.07677
0.1	8	990	0.9741	0.004077	-0.06914	0.1162
0.1	9	1500	0.8070	-0.01352	0.1062	0.1972
0.05	4	495	0.9510	-0.04138	-0.01124	0.1553
0.05	5	65	0.9028	-0.003379	0.02807	0.18607
0.05	6	95	0.9244	0.05050	0.007390	0.1951
0.65	6	145	0.8819	0.03280	0.06871	0.1385
0.05	10	195	0.9123	-0.06046	0.06989	0.1145
0.05	8	295	0.8847	0.04733	0.03952	0.1671
0.1	9	195	0.9410	-0.06926	0.04480	0.1248

The optimum tap weights for the forward and the backward filter for the different data rates at various SNR values are given in Table 4.3.

Now the DFE is synthesized with the tap weights as calculated above. The average probability of error at data rates from 150 Mbps to 375 Mbps and for SNR values from 6 dB to 14 dB is determined. These are shown in Table 4.4 and plotted in Fig. 4.2. It is evident from the curve that for 2.155 km fibre a probability of error less than 10^{-7} can be achieved at higher SNR values. Table 4.4 also shows the total number of symbols processed. As discussed in Chapter 2 the error propagation effect is less as the SNR increases. In Fig. 4.3 we have reproduced the error occurance during the simulation for SNR values of 6,8,10 and 12 dB. These are a sequence of '.' and '*'. '.' indicates a correct decision and '*' indicates an error. We notice that error bursts are more at higher data rates and lower values of SNR. If a group of errors is followed by correct decisions and the number of errors is equal to the number of backward filter taps then a particular error burst can be considered to be over. Any subsequent errors are due to noise and ISI arising from the future samples.

4.2.2 Source-LED, Fibre Length = 2.155 km:

In this case the source used is LED. Therefore the

Table 4.3 : Tap weights for source-Laser, link length =2.155 km

Data Rate in M bit/s.	SNR (dB)	Forward Taps F_o	Forward Taps F_1	Feedback taps B_1	Feedback taps B_2	Feedback taps B_3	Feedback taps B_4	Feedback taps B_5
150	6 dB	0.8181		0.1151	0.04846		-0.04202	
150	8 "	0.8384		0.1722	0.07228		-0.006693	
150	10 "	0.9028		0.1807	0.02807		-0.003379	
150	12 "	0.9696		0.2088	-0.007343		0.014380	
150	14 "	0.9334		0.1600	0.08672		0.03629	
225	6 "	0.8248	0. 08305	0.2959	0.1085	0.02918	0.02825	
225	8 "	0.8342	0.03947	0.2683	0.08387	0.01732	0.03678	
225	10 "	0.9280	-0.01288	0.3055	0.08678	0.04944	-0.01899	
225	12 "	0.9598	0.1066	0.3710	0.1028	0.07664	0.01900	
225	14 "	1.0220	0. 6068	0.3472	0.1270	0.04489	0.08885	
300	6 "	0.8104	0.07912	0.2600	0.1496	0.06675	0.06519	-0.07089
300	8 "	0.8051	0.03392	0.3123	0.09820	0.03289	0.05806	0.01111
300	10 "	0.9410	0.01419	0.4793	0.1620	0.1378	0.06126	0.01673
300	12 "	0.9690	0.04090	0.4836	0.1620	0.1371	0.01272	0.02785
300	14 "	0.9646	0.05837	0.4735	0.1987	0.1071	0.2061	0.03698
375	12 "	0.9151	0.01268	0.5090	0.3192	0.1052	0.01514	0.05308
375	14 "	1.009	-0.08708	0.5688	0.1999	0.1407	0.1598	0.1165
375	24 "	1.022	-0.09712	0.5975	0.2707	0.1233	0.09689	0.04913

Table 4.4: System Performance (Source-Laser, link length = 2.155 km)

Data Rate M bit	SNR (dB)	No. of symbol processed	Error due to ISI only	Error without equalizer	Error	P(e)
150	6	50,000	0	1542	1202	2.40×10^{-2}
	8	50,000	0	551	313	6.26×10^{-3}
	10	10^5	0	269	86	8.6×10^{-4}
	12	2×10^5	0	72	3	1.5×10^{-5}
	14	8×10^6	0	168	0	Better than 1.25×10^{-7}
225	6	50,000	0	2812	1271	2.54×10^{-2}
	8	50,000	0	1645	390	7.8×10^{-3}
	10	10^5	0	1532	106	1.06×10^{-3}
	12	10^5	0	564	3	3×10^{-5}
	14	10^6	0	1542	1	1×10^{-6}
300	6	50,000	0	4836	1954	3.9×10^{-2}
	8	50,000	0	3627	519	1.038×10^{-2}
	10	10^5	0	5304	130	1.3×10^{-3}
	12	10^5	0	3780	13	1.3×10^{-4}
	14	10^6	0	17511	11	1.1×10^{-5}
375	12	10^5	5536	8792	21	2.1×10^{-4}
	14	10^5	5460	7977	2	2×10^{-5}

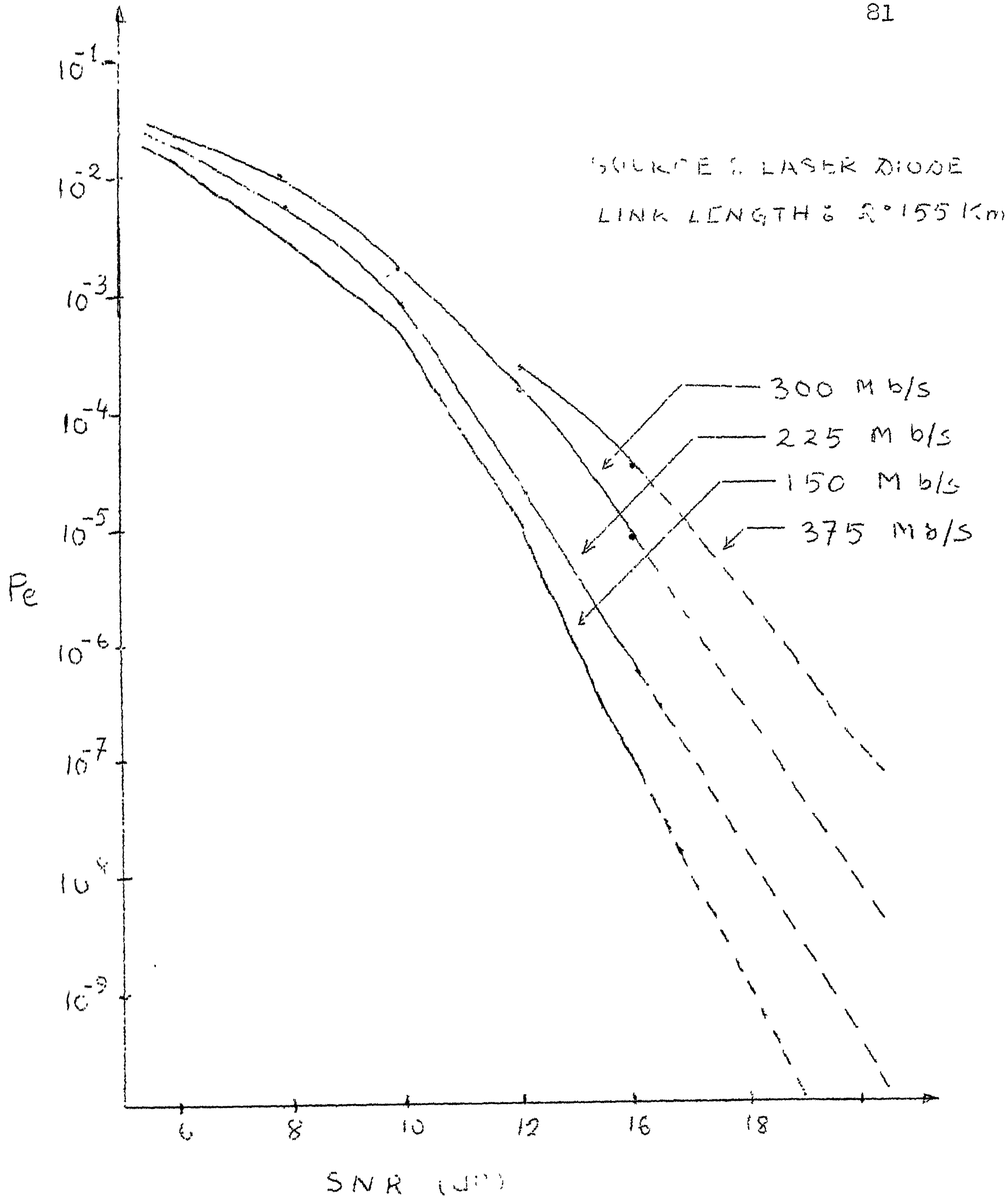


FIG 4.2 DFE PERFORMANCE

FORWARD TAPS

BACKWARD TAPS

FIG. 4.3

SNR=[illegible] FREQ=[illegible] HZ [illegible]

FORWARD TAPS — BACKWARD TAPS

0.8051E+00	0.3397E-01	[illegible]	[illegible]	[illegible]	0.992[illegible]E-[illegible]	[illegible]
0.8051E+00	0.3397E-0[illegible]	[illegible]	[illegible]	[illegible]	0.992[illegible]E-[illegible]	[illegible]

NO OF ERROR [illegible]

NUMBER OF ERROR [illegible] [illegible] , [illegible] OF ERROR IN I/P-O/P [illegible]=95 [illegible]

NUMBER OF ERROR [illegible] [illegible] , [illegible] OF ERROR IN I/P-O/P [illegible]=15 [illegible]

SNR=[illegible] [illegible] [illegible]

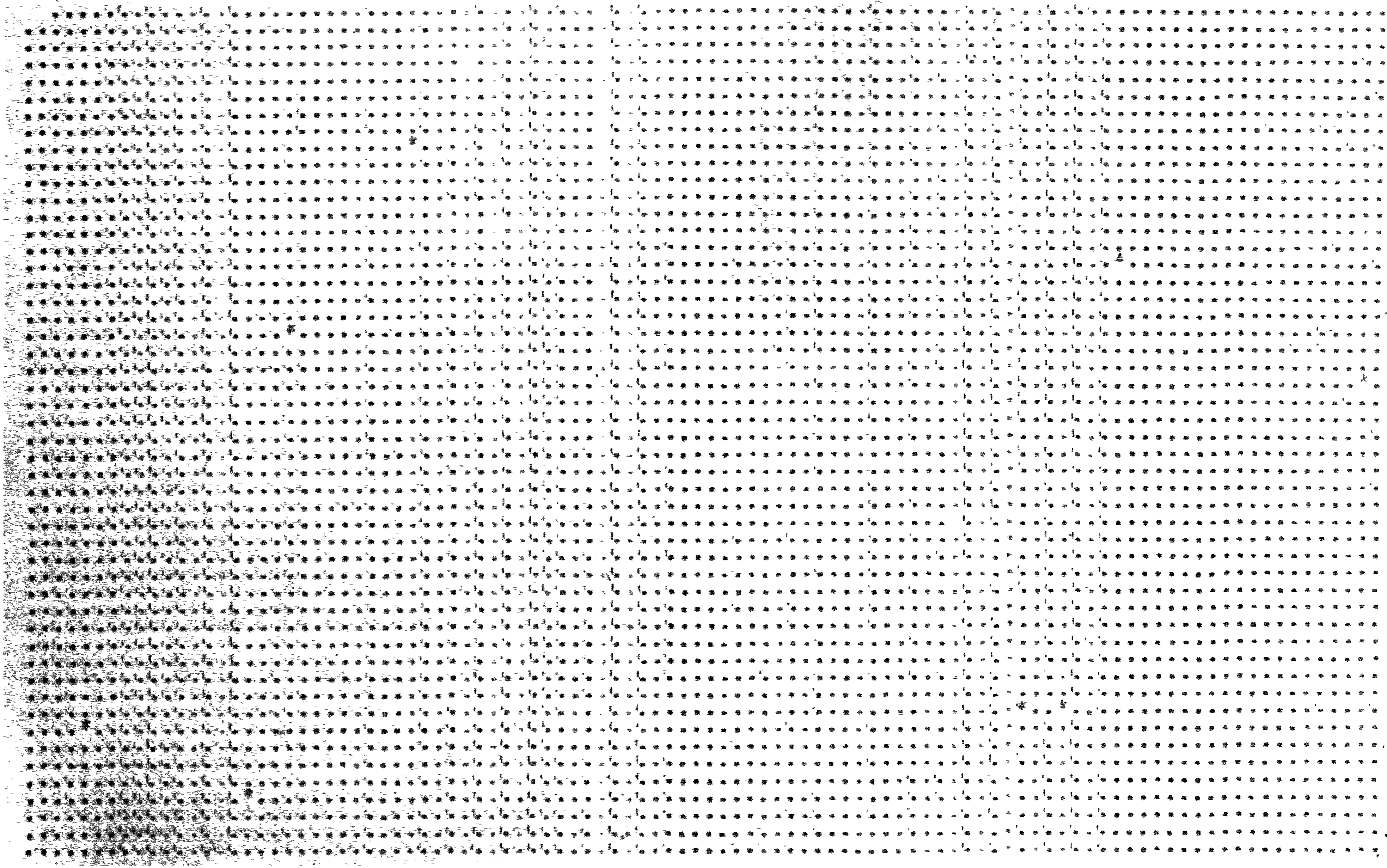
FORWARD TAPS — BACKWARD TAPS

0.94102+00 [illegible] [illegible] [illegible] [illegible] 0.1522E+[illegible] [illegible]
0.9410E+[illegible] [illegible] [illegible] [illegible] .1374E+[illegible] [illegible] [illegible]

NO OF ERROR [illegible]

NUMBER OF ERROR [illegible], [illegible] OF ERROR IN 1/2-3/2 [illegible]=13 VALUE OF K= [illegible]

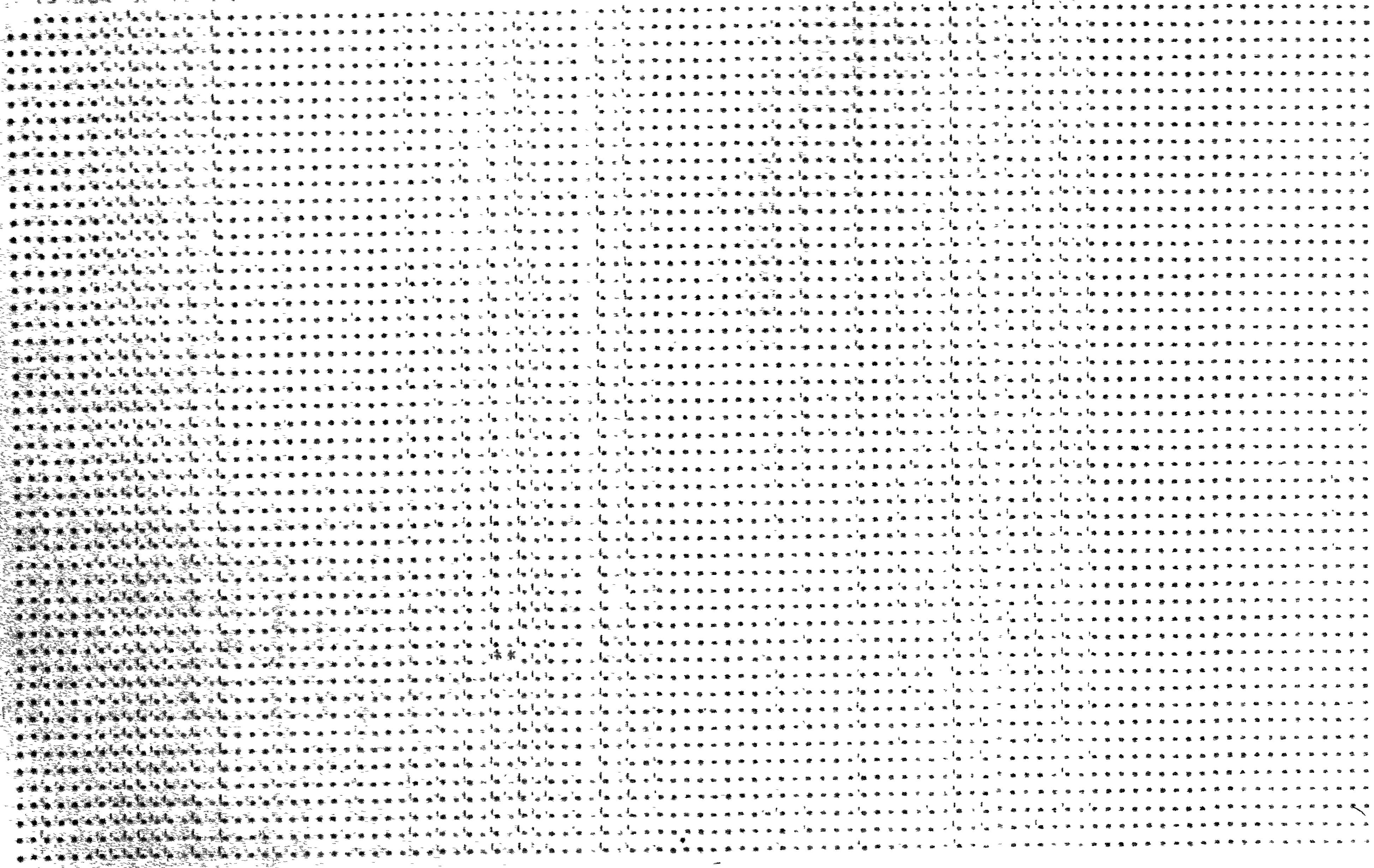
NUMBER OF ERROR [illegible], [illegible] OF ERROR IN 1/2-3/2 [illegible]= 3 VALUE OF K=[illegible]



SNR=12 FREQ=3[illegible] [illegible] [illegible]

FORWARD TAPS BACKWARD TAPS

0.0185E+00 0.4[illegible]E-[illegible] [illegible] [illegible] [illegible] 0.2811E+[illegible] [illegible]
0.0185E+00 0.[illegible]E-[illegible] [illegible] [illegible] [illegible] 0.2811E+[illegible] [illegible]
NO OF ERROR LESS [illegible]
NUMBER OF ERROR [illegible] [illegible] TN T/2-3/2 [illegible] [illegible]
NUMBER OF ERROR [illegible] [illegible] TN T/2-3/2 [illegible] [illegible]



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material dispersion due to the source is much more and it spreads the impulse response further in time domain. The resultant impulse response of the system is shown in Fig. 3.8. In this case also first we determine the optimum tap weights for SNR values of 10,12 and 14 dB. The optimum tap weights are given in Table 4.5. Now the probability of error is found by using the tap gain values as determined above. Probability of error for SNR values 10,12 and 14 dB are plotted in Fig. 4.4. It is clear from the figure that the probability of error less than 10^{-7} can be achieved at higher SNRs and lower data rates. Table 4.6 shows the total number of symbols processed, the number of symbols received in error and the probability of error.

In Fig. 4.5 we have reproduced the system output photograph with errors. These are again a sequence of '.' and '*'. We observe that the error propagation effect due to backward filter is very less.

For the system under discussion we have also carried out simulation at reduced data rates for SNR values of 10,12 and 14 dB, when the equalizer has not been used. In this case, the output of the pre-amplifier is fed directly into the decision circuit. It is seen that at very low data rates we can dispense with the equalizer as the symbol interval is very large and there is much less ISI. Table 4.7 shows the

Table 4.5: Tap weights for Source-LED, link length = 2.155 km

Data Rate in M bits/s	SNR (dB)	Forward Taps F_o	F_1	F_2	Feedback Taps B_1	B_2	B_3
100	10	0.9596			0.2139	0.02134	-0.03313
	12	0.9386			0.1692	0.02214	0.009726
	14	0.9515			0.1993	0.04272	-0.05206
125	10	0.9512	-0.1035	-0.009650	0.2595	0.09460	0.02047
	12	0.9953	-0.09263	-0.02762	0.3505	0.09309	0.01794
	14	0.9476	-0.1306	0.03417	0.3089	0.07797	-0.03819
150	10	0.9291	-0.1766	0.003023	0.3534	0.1334	0.04911
	12	1.009	-0.2240	0.04405	0.4087	0.1393	0.03268
	14	1.059	-0.3001	0.01427	0.4083	0.09660	0.08317
175	10	0.9696	-0.2336	0.04284	0.3560	0.1337	0.05107
	12	1.009	-0.2651	0.003807	0.4180	0.1952	0.09220
	14	0.9511	-0.2186	-0.01559	0.3616	0.1316	0.07032

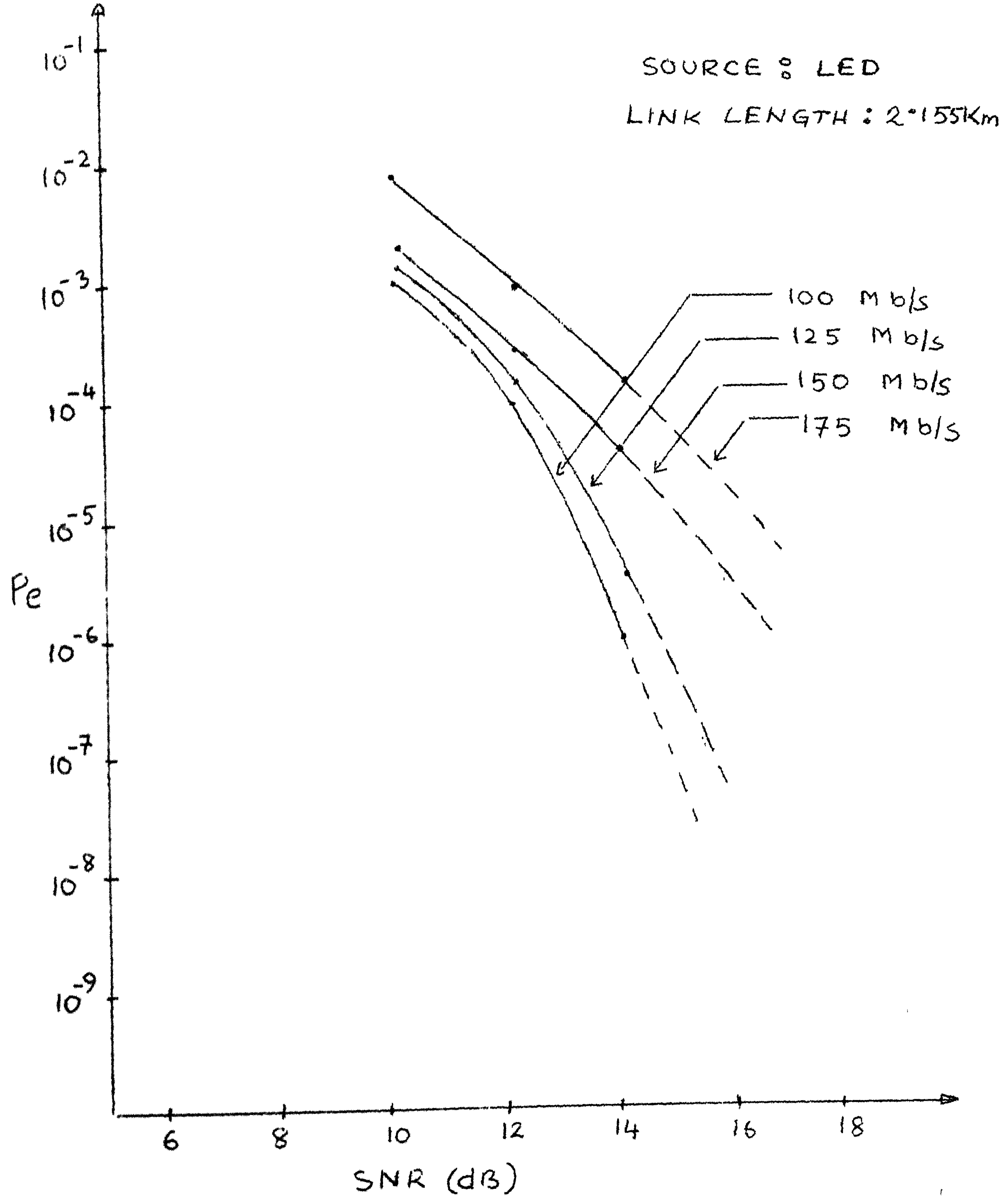

FIG 4.4 DFE PERFORMANCE

Table 4.6: System Performance (Source-LED, link length = 2.155 km)

Data Rate in M bits/s	SNR (dB)	No.of symbol processed	Error due to ISI only	Error without equaliser	Error	P(e)
100	10	70,000	0	410	91	1.3×10^{-3}
	12	10^5	0	122	11	1.1×10^{-4}
	14	2×10^6	0	266	4	2×10^{-6}
125	10	50,000	0	814	96	$1.92\times10^{-[illegible]}$
	12	1×10^5	0	332	18	$1.80\times10^{-[illegible]}$
	14	2×10^6	0	5845	9	4.5×10^{-6}
150	10	50,000	0	2127	180	3.6×10^{-3}
	12	10^5	0	2962	42	4.2×10^{-4}
	14	10^6	0	20551	25	2.5×10^{-5}
175	10	50,000	4651	4093	458	9.16×10^{-3}
	12	50,000	4687	3807	44	8.8×10^{-4}
	14	10^5	9337	7082	23	2.3×10^{-4}

SNR=1[illegible] [illegible] (LED)

FORWARD TAPS — BACKWARD TAPS

0.9696E+00 -0.2[illegible] [illegible] 0.355[illegible]
0.9696E+00 -0.2335[illegible] [illegible] 0.355[illegible]
NUMBER OF [illegible] IN 1/2-3/2 ARE= [illegible]
NUMBER OF [illegible] IN 1/2-3/2 ARE= [illegible]
OF ERROR LESS [illegible]

SNR=12 [illegible] (STEP SIZE)= [illegible] (LED)

FORWARD TAPS — BACKWARD TAPS

0.4017E+01 [illegible] [illegible] [illegible] [illegible] [illegible]
0.4009E+01 [illegible] [illegible] [illegible] [illegible] [illegible]
NO OF ERROR [illegible]
NUMBER OF ERROR [illegible] ARE=7[illegible], [illegible] I/P=3/2 ARE=1[illegible] [illegible]
NUMBER OF ERROR [illegible] ARE=371, [illegible] I/P=3/2 ARE= 5 [illegible] <=5

Table 4.7: System Performance without DFE (Source-LED, link length = 2.155 km)

Data Rate in M Bits/s	SNR (dB)	No. of Symbol processed	Error	P(e)
75	10	10,000	32	3.2×10^{-3}
	12	50,000	9	1.8×10^{-4}
	14	5,10,205	5	9.8×10^{-6}
62.5	10	10,000	9	9×10^{-4}
	12	10^{5}	10	1×10^{-4}
	14	4×10^{6}	5	1.25×10^{-6}
50	10	20,000	15	7.5×10^{-4}
	12	111,762	5	4.47×10^{-5}
	14	5×10^{6}	1	2×10^{-7}

data rate, number of symbols processed and the probability of error without the equalizer.

4.2.3 Source-Laser, Fibre Length = 4.310 km:

In the system under discussion we assumed two equal fibre sections of length 2.155 km. each have been ideally connected to one another. For simulation the impulse response of the system as shown in Fig.3.11 is considered. Again for this system first we determined the optimum tap weights of the forward and backward filters by the adaptive method for SNR values of 10,12 and 14 dB. The optimum tap weights are given in Table 4.8.

The average probability of error at different data rates for 4.310 km fibre length and for various SNR values is determined. The probability of error for various SNRs is plotted in Fig. 4.6. It is observed that with the increase of the link length the probability of error for the same data rate and SNR is higher. Table 4.9 shows the number of symbols processed, the number of errors in simulation and the probability of error.

For this system also we have carried out simulation at much reduced data rates when the equalizer is not used. The results are shown in Table 4.10. The main interference is due to the noise. Therefore, the system works better without any equalizer at higher values of SNR.

Table 4.8: Tap weights for Source-Laser, link length = 4.310 km

Data Rate in Mbits/s	SNR (dB)	Forward Taps F_o	F_1	Feedback Tap B_1	B_2	B_3	B_4	B_5
150	10	0.8969		0.3153	0.1062	0.01517	-0.01405	
	12	0.9357		0.3317	0.1250	0.03097	0.01873	
	14	0.9791		0.4004	.09755	0.01817	-0.007171	
200	10	0.9046	0.02048	0.4635	.2255	0.05147	0.02256	
	12	0.9352	0.002023	0.4585	0.2652	0.06820	0.006570	
	14	0.9796	-0.002269	0.4935	0.3559	0.08362	-0.05300	
200	10	0.9310	-0.01488	0.5687	0.3520	0.2413	0.06583	0.02405
	12	0.9672	-0.04332	0.5730	0.2841	0.2283	0.04145	-0.02053
	14	0.9973	-0.04714	0.5528	0.2913	0.1870	-0.02107	-0.01404

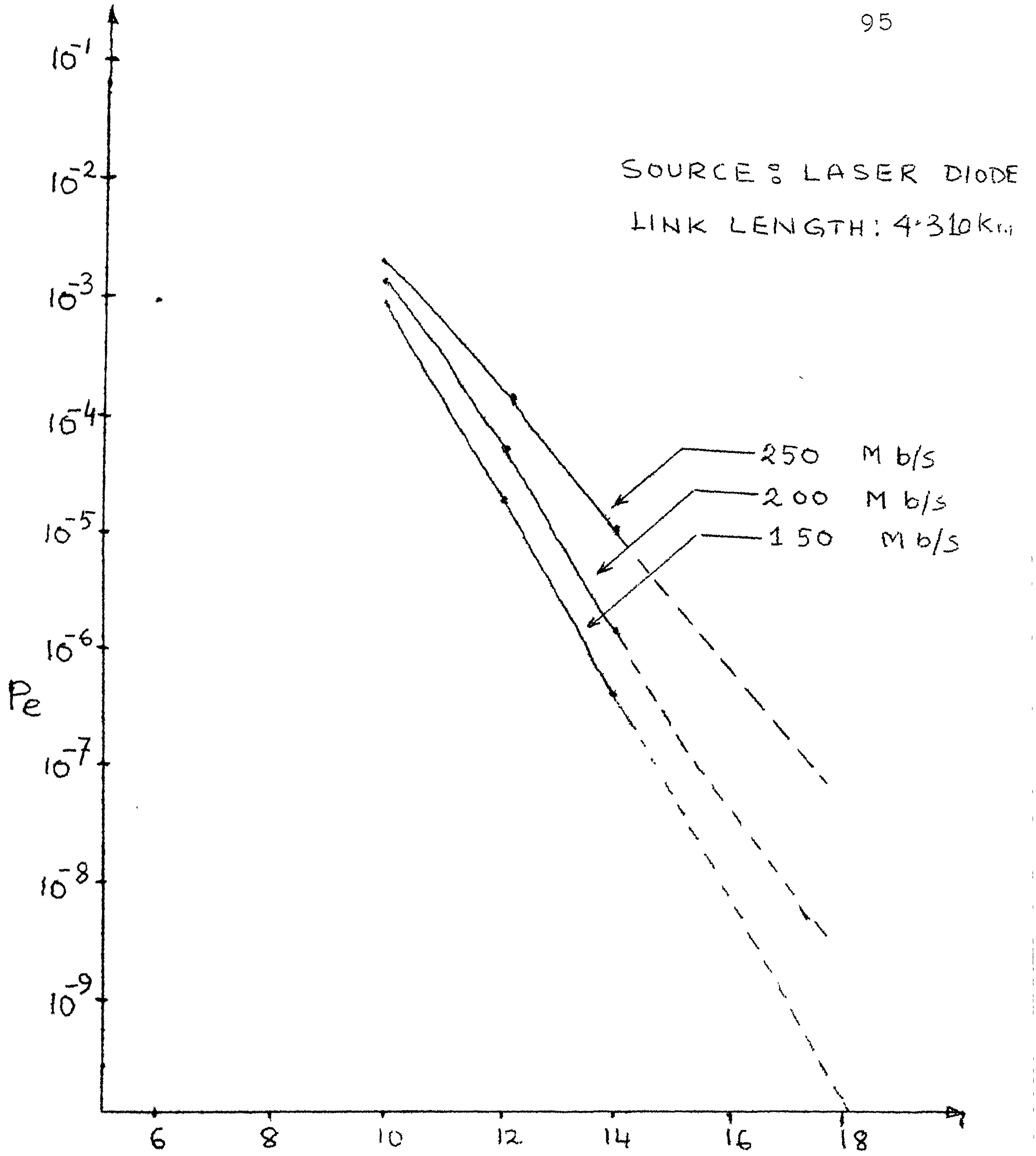

FIG 4.6 DFE PERFORMANCE

Table 4.9: System Performance (Source-Laser, link length = 4.310 km)

Data Rate in M Bits/s	SNR (dB)	No. of symbol processed	Error due to ISI only	Error without equalizer	Error	P(e)
150	10	10^5	0	1704	109	1.09×10^{-3}
	12	2×10^5	0	1294	5	2.5×10^{-5}
	14	5×10^6	0	9355	3	6×10^{-7}
200	10	10^5	0	6428	121	1.21×10^{-3}
	12	10^5	0	4934	6	6×10^{-5}
	14	2×10^6	0	76110	3	1.5×10^{-6}
250	10	50,000	4690	5640	152	3.04×10^{-3}
	12	50,000	4585	5240	7	1.4×10^{-4}
	14	10^6	93772	102037	11	1.1×10^{-5}

Table 4.10: System Performance Without DFE
(Source-Laser, link length = 4.310 km)

Data Rate in M bits/s	SNR (dB)	No. of Symbol Processed	Err	P(e)
100	10	10,000	69	6.9×10^{-3}
	12	10,000	14	1.4×10^{-3}
	14	10^{5}	9	9×10^{-5}
87	10	20 000	43	2.15×10^{-3}
	12	20000	9	4.5×10^{-4}
	14	10^{5}	8	8×10^{-5}
75		20,000	31	1.55×10^{-3}
	,2	10^{5}	16	1.6×10^{-4}
	14	10^{6}	2	2×10^{-6}
60	10	20,000	30	1.5×10^{-3}
	12	46,300	5	1.08×10^{-4}
	14	10^{6}	0	Better than 10^{-6}

CHAPTER 5

CONCLUSION

This thesis has been directed to evaluate the performance of some fibre optic digital communication systems. We have shown an optical fibre system with its essential components. The choice of source and photodetector depends upon the length of the fibre, attenuation of the fibre and desired probability of error for a given data rate. We have seen that to use the same fibre length at higher data rates, one has to use a laser diode as the source, as the power required to combat the ISI penalty is not sufficient when we use a LED. Similarly the increase in fibre length limits the data rate which can be transmitted over the link for a desired P(e).

We have considered systems with different sources and fibre lengths as follows.

a) Source-Laser diode, link length = 2.155 km

b) Source-LED, link length = 2.155 km

c) Source-Laser diode, link length = 4.310 km

We have noticed that the fibre channel spreads the optical signal in time domain. This dispersion and additive noise of the system cause ISI. The effect of ISI can be reduced with an equalizer.

In this thesis we have designed a DFE based on the Monsen's Gradient-technique. DFE uses both the received signal and reconstructed data to form its decision. The tap weight values for the DFE have been determined by Monsen's adoptive linear filtering method.

To determine the tap weight values and to evaluate the performance of the system, we carried out simulation on the digital computer DEC-1090 system. Optimum tap values are obtained when the number of taps of the forward filter is equal to the number of future ISI sample plus one and taps are spaced at the symbol interval.

The initial aim was to find the maximum data rate and SNR value for a system which gives a P(e) of 10^{-8}. To get the P(e) of 10^{-8}, one has to generate and process on an average 10^{9} noise and data symbols. From simulation we have noticed that to generate and process 10^{6} symbols on the computer CPU and execution time are approximately 6.8 and 35 minutes, respectively. Therefore, a maximum of 8×10^{6} symbols could be generated and processed in the simulation.

The fundamental assumption in the design of the DFE is that the decisions are error free, but, in every simulation certain amount of error, i.e. 'Error Bursts'

have occured. Any wrong decision affects the decision in the DFE till it propo[illegible] in the feedback filter. Error bursts are more at higher data rates and lower SNR values.

From the results we have noticed that for a data rate of 150 M bits/sec., the system with the LED source gives a probability of error of 2.5×10^{-5} for a SNR of 14 dB. Whereas, with the laser diode a probability of error of 1.25×10^{-7} is obtained for the same SNR value. Also from the extension of the performance curve (Fig. 4.2) for 150 M bits/sec. it is observed that a P(e) of 10^{-9} for the fibre length of 2.155 km can be achieved at an approximate SNR of 16 dB. Whereas for the fibre length of 4.31O km (Fig. 4.6), it can be achieved at approximately 18 dB.

For systems (b) and (c) performance of the links have also been evaluated without an equalizer. For source-LED and a link length of 2.155 km, it is observed that to achieve the same P(e) at a SNR of 14 dB, the maximum data rate which can be transmitted without an equalizer is approximately 60% of the data rate obtained with the equalizer (Ref. Table 4.6 and 4.7).

For the source-laser and link length of 4.31 km, to achieve the same P(e) at a SNR of 14 dB, the maximum

data rate which can be transmitted without an equalizer is less than 50% of the data rate obtained with the equalizer (Ref. Table 4.9 and 4.10). From this we observe that with the equalizer, we can transmit atleast double the data rates than without the equalizer to achieve the same P(e).

Capt. R. Wahi [15] designed a DFE based on Austin's sub-optimal method. He has used a data rate of 10 M-bits/sec. and a fibre whose impulse response at 10% amplitude has a full width of 275 ns. For 14 dB SNR he has obtained a P(e) of 3×10^{-5}. In our study we have used a resultant impulse response, whose width at 10% amplitude is 10.5 ns. We have used data rates from 150 to 375 M-bits

In our case for a SNR of 14 dB, the P(e) at 300 and 375 M-bits/sec. is 1.5×10^{-5} and 2×10^{-5}, respectively. For 150 M-bit/sec. and SNR of 14 dB, we get P(e) better than 1.25×10^{-7}. In Capt. R. Wahi's study the ratio of full width at 10% to symbol interval is 2.75 and in our study it is 3.18 and 3.9 for data rates 300 and 375 M bits/sec, respectively. From the above this system seems to be better.

One could study the performance of DFE after actual measurements of the overall impulse response of the system and then simulating on the computer.

Another issue for further study is the simulation of the entire system including line coder, bit synchronizer and decoder etc., on the digital computer. The performance of the system can also be studied after hardware implementation of the DFE and then its performance can be compared with the simulation results.

Finally system can also be investigated by considering the affect of shot noise on the system.

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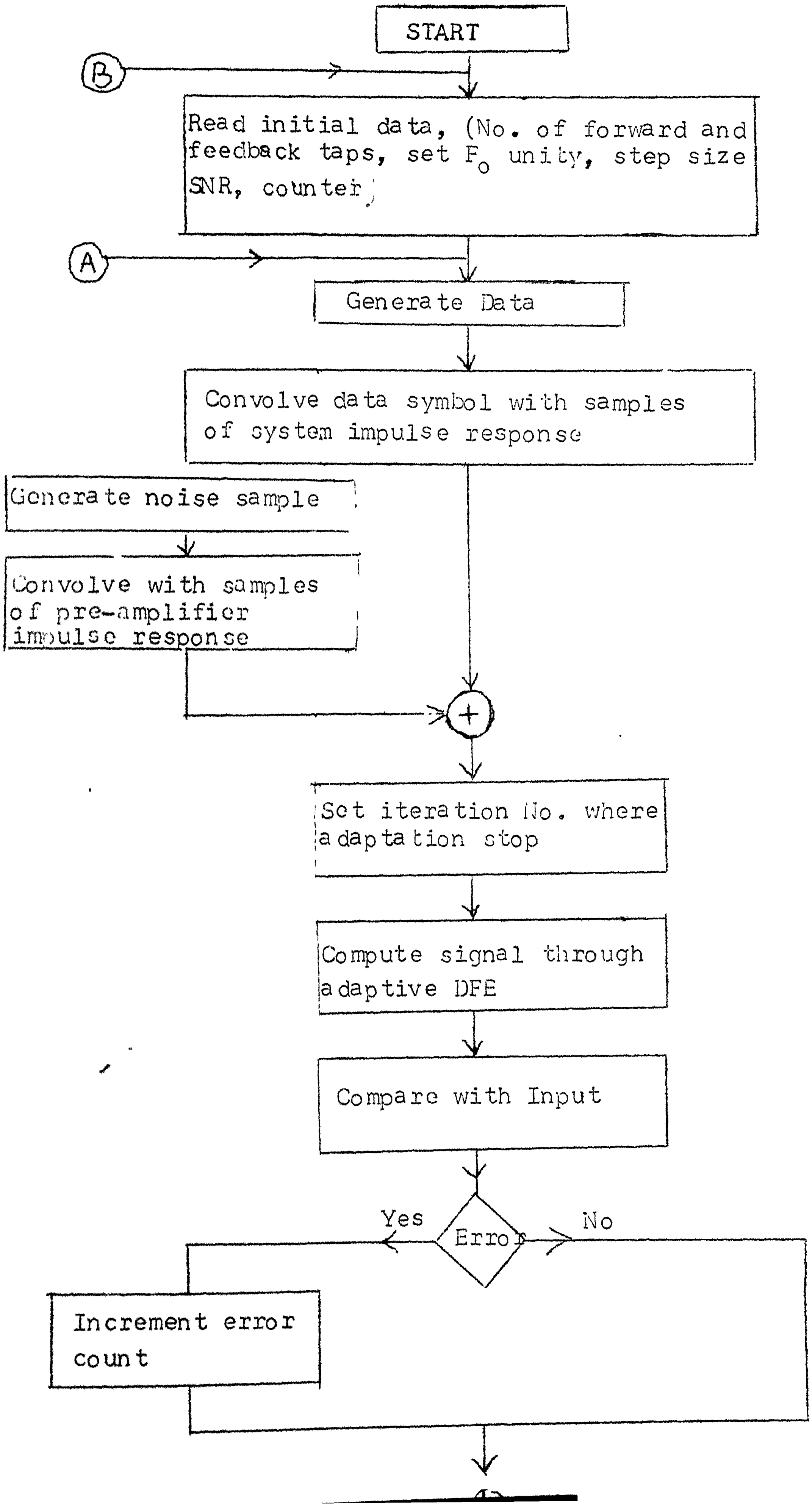
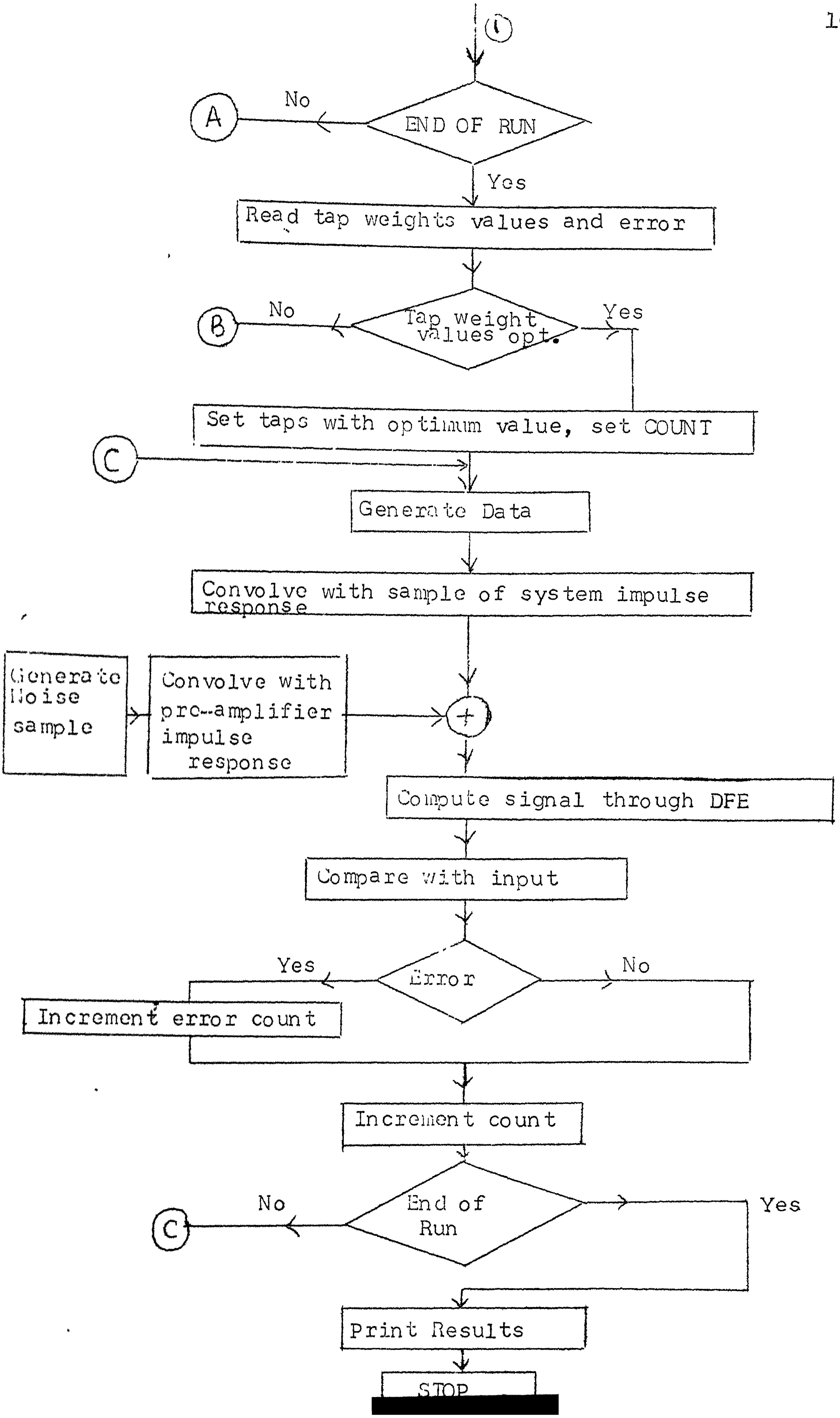
APPENDIX

FLOW CHART




(i)
END OF RUN
No
A
Yes
Read tap weights values and error
Tap weight values opt.
No
B
Yes
Set taps with optimum value, set COUNT
C
Generate Data
Convolve with sample of system impulse response
Generate Noise sample
Convolve with pre-amplifier impulse response
+
Compute signal through DFE
Compare with input
Error
Yes
No
Increment error count
Increment count
End of Run
No
C
Yes
Print Results
STOP

PROGRAM LISTING

```
C       THIS PROG CALCULATE FORWARDIS  & BACKWARD TAP GAINS
C       AND EVALUATE THE PERFORMANCE OF DECISION FEEDBACK EQUALISER
        DIMENSION T(15),F(15),S(15),B(15),P(5),GN(5),H(15),X(15)
        DIMENSION A(100),FF(50),BB(50),D(100000)
        REAL  MAK
        OPEN(UNIT=15,DEVICE='DSK',FILE='TAP2.DAT')
        OPEN(UNIT=16,DEVICE='DSK',FILE='TAP22.DAT')
C       SNR=10      FREQ=150 MHZ
        WRITE(15,92)
92      FORMAT(20X,'SNR=10   FREQ=150 MHZ    ALFA(STEP SIZE)=0.05 ',/20X
        1 '--------------------------------------------------------'///)
C       ALFA=STEP SIZE
        ALFA=0.05
        ALFAN=0.0
C       SIGMA=STANDARD DEVIATION(FOR SNR)
        SIGMA=0.199526
C       P(T)=PRE-AMPLIFIER IMPULSE RESPONSE SAMPLE VALUES
        P(1)=1.000;P(2)=0.0339
        GN(1)=0.0  ;GN(2)=0.0 ;GN(3)=0.0  ;GN(4)=0.0;GN(5)=0.0
C       H(T)=SYSTEM IMPULE RESPONSE SAMPLE VALUES
        H(1)=0.0973;H(2)=1.0;H(3)=0.6007;H(4)=0.3123;H(5)=0.2102
        H(6)=0.0627;H(7)=0.0190
        X(1)=0.0;X(2)=0.0;X(3)=0.0;X(4)=0.0;X(5)=0.0;X(6)=0.0;X(7)=0.0
        X(8)=0.0;X(9)=0.0;X(10)=0.0;X(11)=0.0;X(12)=0.0;X(13)=0.0
C       T(T)=LOCATION OF SIGNAL IN FORWARD FILTER
        T(1)=0.0;T(2)=0.0000; T(3)=0.0000;T(4)=0.0000;T(5)=0.0000
        T(8)=0.0;T(9)=0.0; T(10)=0.0
C       F(T)=FORWARD FILTER TAP VALUE
        F(1)=1.0000;F(2)= .0000 0;F(3)=0.0000000;  F(4)=0.0;F(5)=0.0
        F(7)=0.0;F(8)=0.0;F(9)=0.0;F(10)=0.0
C       S(T)=LOCATION OF BACKWARD FILTER FOR RECONSTRUCTED DATA
        S(1)=0.0;S(2)=0.0 ;S(3)=0.0 ;S(4)=0.0;S(5)=0.0;S(6)=0.0;S(7)=0.0
        S(8)=0.0;S(9)=0.0;S(10)=0.0;S(11)=0.0
C       B(T)=FEEDBACK FILTER TAP VALUE
        B(1)=0.000000; B(2)=0.0000 ;B(3)=0.000000;B(4)=0.0000;B(5)=0.0000
        B(6)=0.0    ;B(7)=0.0000
```

```
C
C         JP=NUMBER OF SAMPLES OF PRE-AMPLIFIER IMPULSE RESPONSE
C         JH=NUMBER OF SAMPLES OF SYSTEM IMPULSE RESPONSE
C         JF=NUMBER OF FORWARD FILTER TAPS
C         JB=NUMBER OF FEEDBACK FILTER TAPS
C         IFS=NUMBER OF FUTURE ISI SAMPLE
C         IFTBM=NUMBER OF FUTURE ISI SAMPLE BEFORE MAIN TAP OF FWD FILTER
C         ITOT=IFS+IFTBM
C         IRANGE=ITERATION No FROM WHERE VALUES OF TAP GAIN STORED
C         JRANGE=ITERATION No UPTO WHICH VALUES OF TAP GAIN STORED
C         IFIX=  ITERATION No AT WHICH ADAPTATION STOP
          JP=2; JH=7 ; JF=2; JB=5; IFS=1;IFTBM=1 ;ITOT=2
          IRANGE= 1493 ; JRANGE= 1497  ; IFIX= 1495
          N=10000
          NP=20
          K=0
          JE1=0
          NOIS=0.9
          NO2=0.0
          CALL G05CCF
          DO  10  I=1,N
          K=K+1
          AK=G05CAF(A)
          IF(AK.GE.0.5)  GO  TO 31
          AK=-1.0
           GO  TO 32
 31       AK=1.0
 32       DO  23  J=1,ITOT
C         A(J)=INPUT DATA TO THE SYSTEM
          A(J)=A(J+1)
 23       CONTINUE
          A(ITOT+1)=AK
C         PUT J=NUMBER ONE LESS THAN THE NO OF H SAMPLE
          DO  33  J=1,JH-1
          X(J)=X(J+1)
 33       CONTINUE
          X(JH)=AK
```

```
C
      VK=0.0
C     NO OF ITERTION EQUAL TO NO OF H SAMPLE
      DO  34   I=1,JH
      IF((I-I).GT.0) GO  TO  29
      VK=VK+H(I)*X(JH+1-I)
34    CONTINUE
C     EQUATE K TO NUMBER OF FUTURE SAMPLE IN COMBINED IMP RESPONSE
29    IF(K.GE.IFS) GO  TO 10
      GK=G05DDF(AMEAN,SIGMA)
C     J IS UPTO NO OF PRE AMP SAMPLE MINUS ONE
      DO  35   J=1,JP-1
      GN(J)=GN(J+1)
35    CONTINUE
        GN(JP)=GK
      ANK=0.0
C     NO OF ITERATION EQUAL TO NO OF PRE-AMP SAMPLE
      DO  36   J=1,JP
      IF((I-IFS-J).LT.0)  GO  TO  30
      ANK=ANK+P(J)*GN(JP+1-J)
36    CONTINUE
30    IF(VK.GE.0)  GO  TO  60
      EK=-1.0
      GO TO 61
60    EK=1.0
61    IF(A(1+IFTBM).EQ.EK)  GO   TO 62
      JE1=JE1+1
C     RK=INPUT SIGNAL TO DEF
62    RK  =VK+ANK
      IF(RK.GE.0)  GO   TO  37
C     SK=RECONSTRUCTED DATA WITHOUT DFE
      SK  =-1.0
      GO TO 38
37    SK  =1.0
38    IF(A(1+IFTBM).EQ.SK  )   GO  TO 40
      NO1S=NO1S+1
40    IF(K.GT.10) GO    TO 41
```

```
C
      WRITE(46,99)( VK,ANK,RK,X(JH),GN(JP),A(1+IFTRM),SK,AK,GK)
99    FORMAT(10X,3(E15.5),5X,2F8.3,2F5.1,5X,A1,F5.1,2X,F6.3)
C     TAP GAIN ITERATION START FROM THIS POINT
C     ------------------------------------------------------------
41    DO 12 J=1,JF-1
      T(J)=T(J+1)
12    CONTINUE
      T(JF)=RK
C     EQUATE K TO NO OF FUTURE SAMPLES +NO OF FWD TAPS BEFORE MAIN TAP
      IF(K.LE.(ITOT)) GO  TO 10
C     GIVE RANGE IN WHICH VALUE OF TAP GAIN REQUIRED
      IF(I.GE.IRANGE)  GO  TO 26
      IF(K.GT.IRANGE)  GO  TO 26
      DO    11    J=1,JF
      FF(J)  =F(J)
11    CONTINUE
      DO   09  J=1,JB
      BB(J)  =B(J)
09    CONTINUE
C     YK=OUTPUT OF FORWARD FILTER
26    YK=0
      DO 13  J=1,JF
      YK=YK+T(J)*F(J)
13    CONTINUE
      DO 14  J=1,JB
      S(J)=S(J+1)
14    CONTINUE
C     ZK=OUTPUT OF FEEDBACK FILTER
      ZK=0
      DO 15  J =1,JB
      ZK=ZK+S(J)*B(J)
15    CONTINUE
      SCURLK=YK-ZK
      IF(SCURLK.GE.0) GO TO 25
      S(JB+1)=-1
```

```
C
C      S(JB+1)=RECONSTRUCTED DATA AFTER DFE
       GO TO 16
25     S(JB+1)=1
16     IF(K.GE.IFIX) GO TO 22
C      IF(K.GE.(ITOT+1))  GO  TO  22
       EK  =(SCURLK-S(JB+1))
C      EK=ERROR SIGNAL
C      USE MIN VALVE OF K AS DEFINED EARLIER PLUS 1 AFTER TAPS ARE FIXED
C      GIVE ITERATION NO TO FIX TAPS GAIN
C      IF(K.GE.5        ) GO TO 22
       EMULTI=EK*ALFA
       DO  17 J=1,JF
C      F(J)=UPDATED TAP GAIN VALUE OF FORWARD FILTER
       F(J)=F(J)-EMULTI*T(J)
C      TYPE*,F(1)
17     CONTINUE
       DO 18 J=1,JB
C      B(J)=UPDATED TAP GAIN VALUE OF FEEDBACK FILTER
       B(J)=B(J)+EMULTI*S(J)
18     CONTINUE
22     IF(S(JB+1).GT.0) GO TO 19
       ICK=-1.0
       GO TO 20
19     ICK=1
20     IF(A(1)  .EQ.ICK)        GO  TO  21
       D(I)='*'
       NO2=NO2+1
       WRITE(45,*)NO2,K
       IF(NO2.GT.100)   GO  TO  70
       GO TO 28
21     D(I)='.'
C      GIVE SAME RANGE TO WRITE THE TAP GAIN VALVES IN O/P FILE
28     IF(K.LE.IRANGE)  GO  TO 27
       IF(K.GT.JRANGE  )  GO TO 27
C      WRITE(38,85)(YK,ZK,SCURLK,S(JB+1),E(K))
       WRITE(45,86)(FF(J),J=1,JF),(BB(J),J=1,JB)
```

```
C                                     112
C       GIVE  TOTAL NUMBER OF FWD &BACKWARD TAPS
C       ----------------------------------------
86      FORMAT(2X,7(E13.4))
27      CONTINUE
10      CONTINUE
70      WRITE(45,91)JE1
91      FORMAT(10X,'NO OF ERROR LESS NOISE=',I7,5X,F5.1)
        WRITE(45,93)NO1S,NO2,K
93      FORMAT(5X,'NUMBER OF ERROR AFTER SLICER ARE=',I6,5X,'NUMBER OF
        1 ERROR IN I/P-O/P ARE=',I4,5X,'VALVE OF K='I7)
        DO   94   I=1,NR
C        WRITE(45,97)((IC(I*25-25+J),IR(I*25-25+J),A(I*25-25+J)),J=1,25)
        WRITE(45,97)(D(I*100-100+J),J=1,100)
94      CONTINUE
97      FORMAT(10X,100(A1))
        WRITE(46,*)JE1
        WRITE(46,*)NO1S
        DO   95   I=1,NR
        WRITE(46,97)(M(I*100-100+J),J=1,100)
95      CONTINUE
        WRITE(45,99)
99      FORMAT(10X,'ERROR IN ITERATION')
        DO 91 I=1,2
        WRITE(45,98)(E(I*10-10+J),J=1,10)
91      CONTINUE
98      FORMAT(10(1X,E12.4))
        STOP;END
```